Wavefront propagation in Uniaxial crystal

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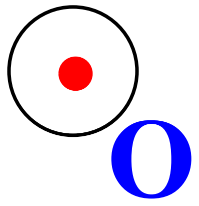
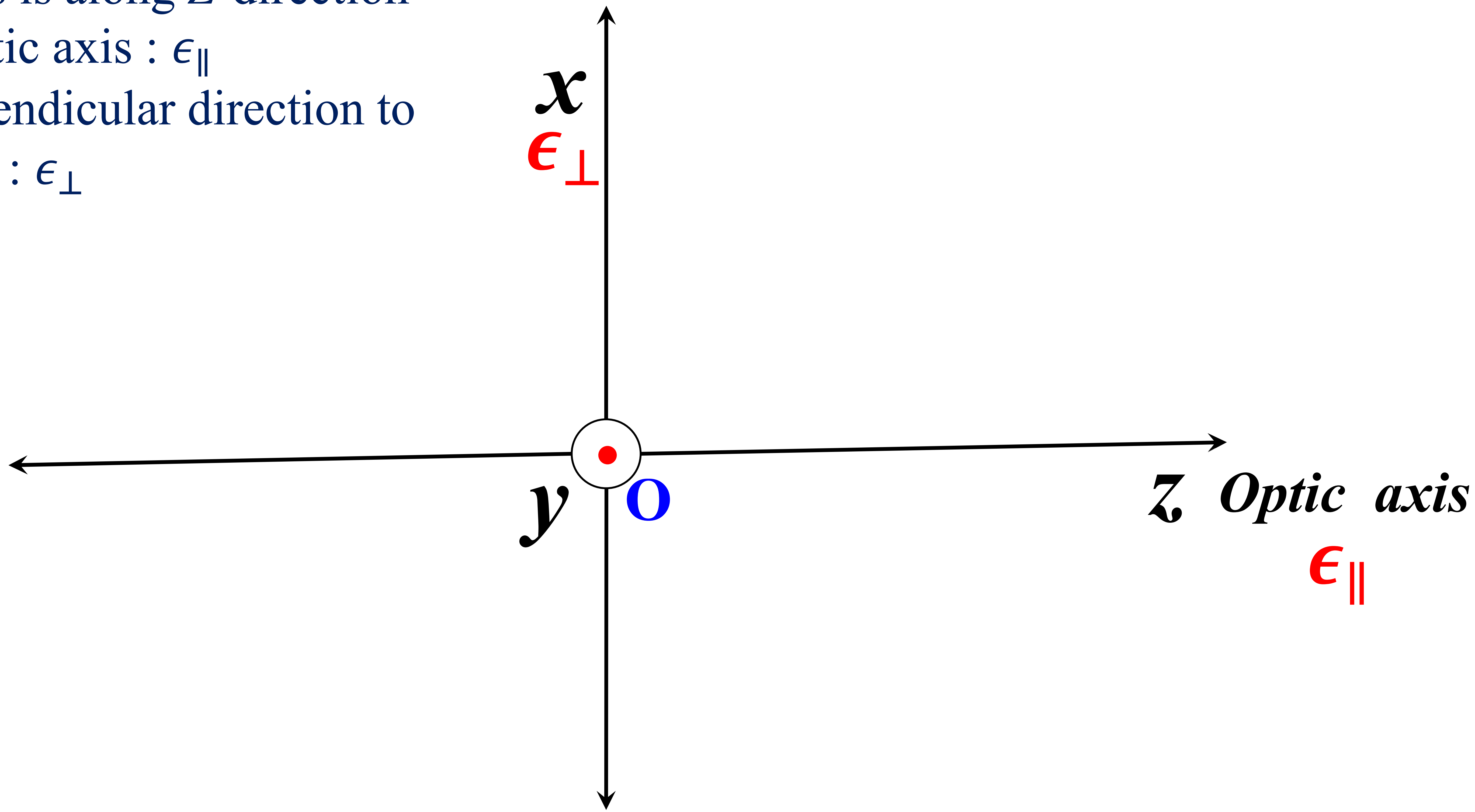


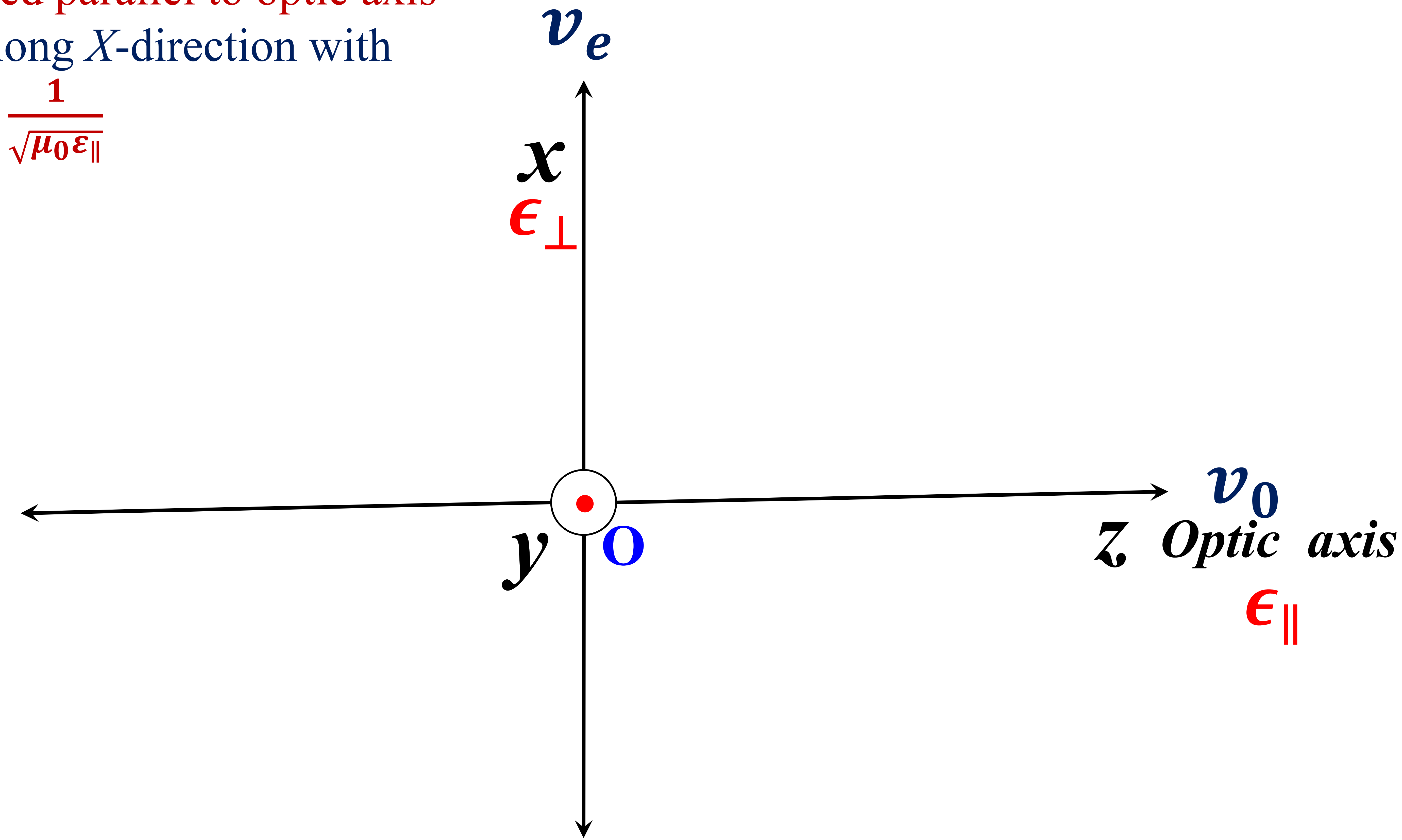
Consider a point source emanating unpolarized light
in a uniaxial crystal

It is the choice of coordinate axes

- Optic axis is along *Z*-direction
- Along optic axis : $\epsilon_{\parallel}$
- Any perpendicular direction to optic axis : $\epsilon_{\perp}$

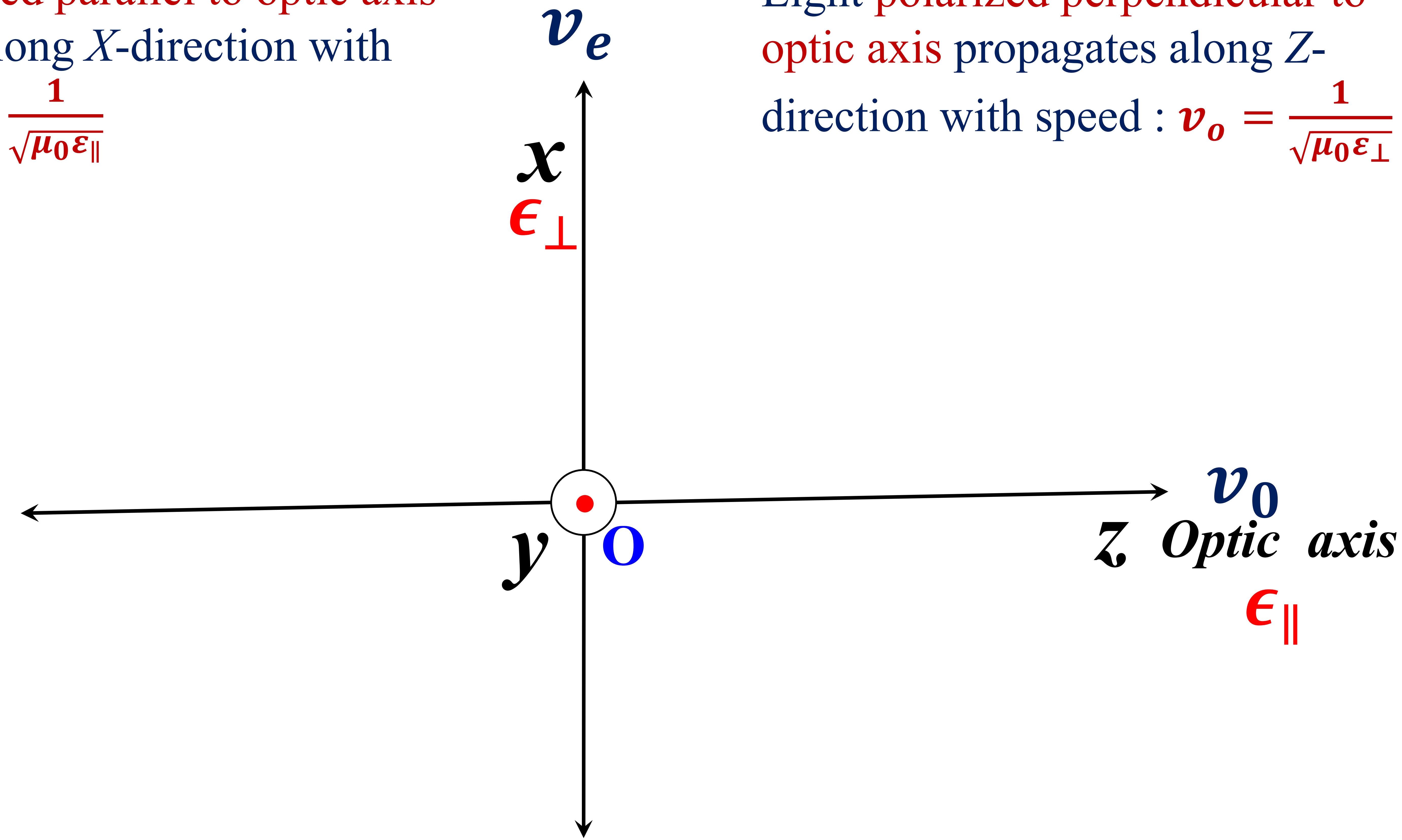


Light polarized parallel to optic axis propagates along X-direction with speed : $v_e = \frac{1}{\sqrt{\mu_0 \varepsilon_{\parallel}}}$

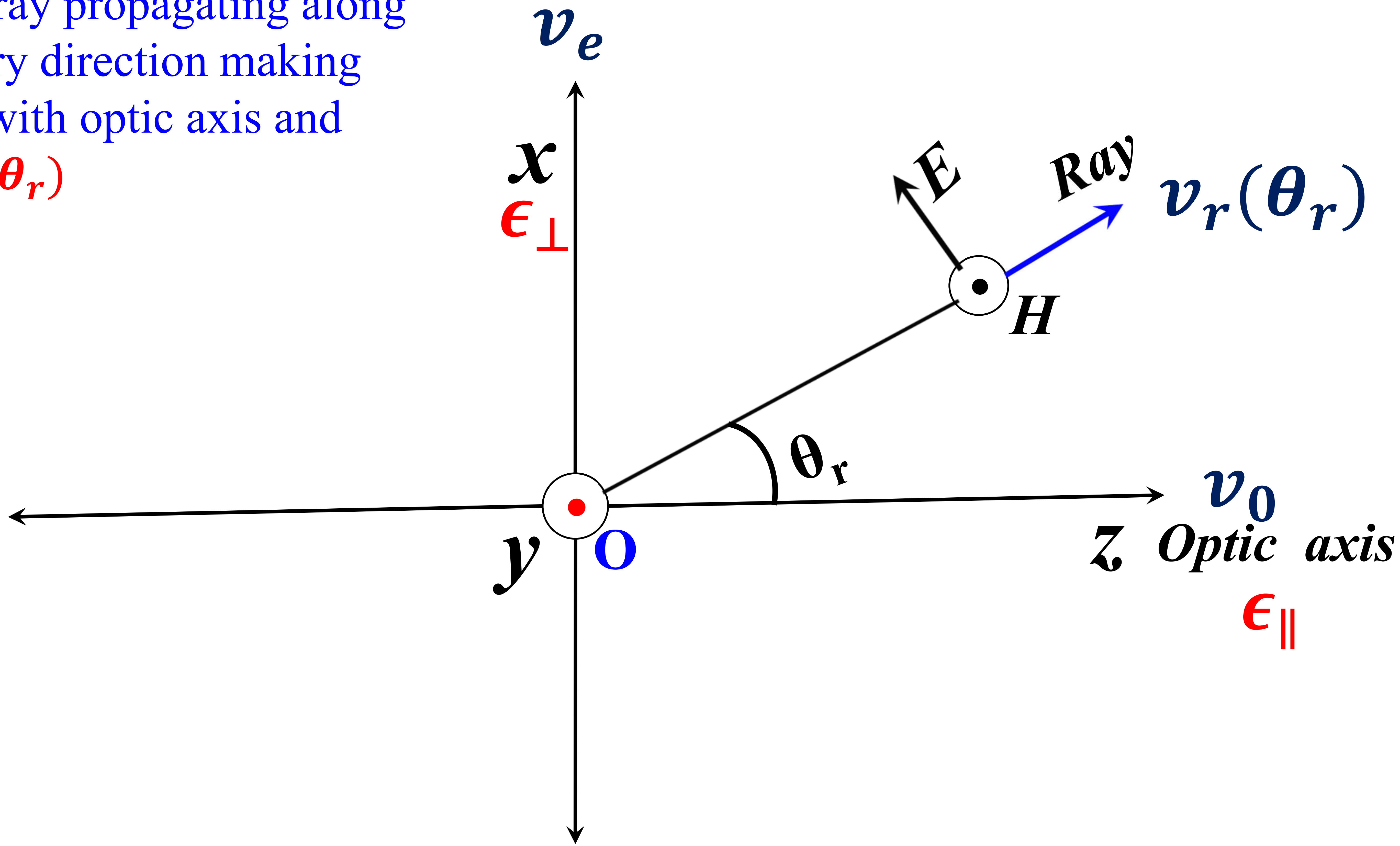


Light polarized parallel to optic axis propagates along *X*-direction with speed : $v_e = \frac{1}{\sqrt{\mu_0 \varepsilon_\parallel}}$

Light polarized perpendicular to optic axis propagates along *Z*-direction with speed : $v_o = \frac{1}{\sqrt{\mu_0 \varepsilon_\perp}}$

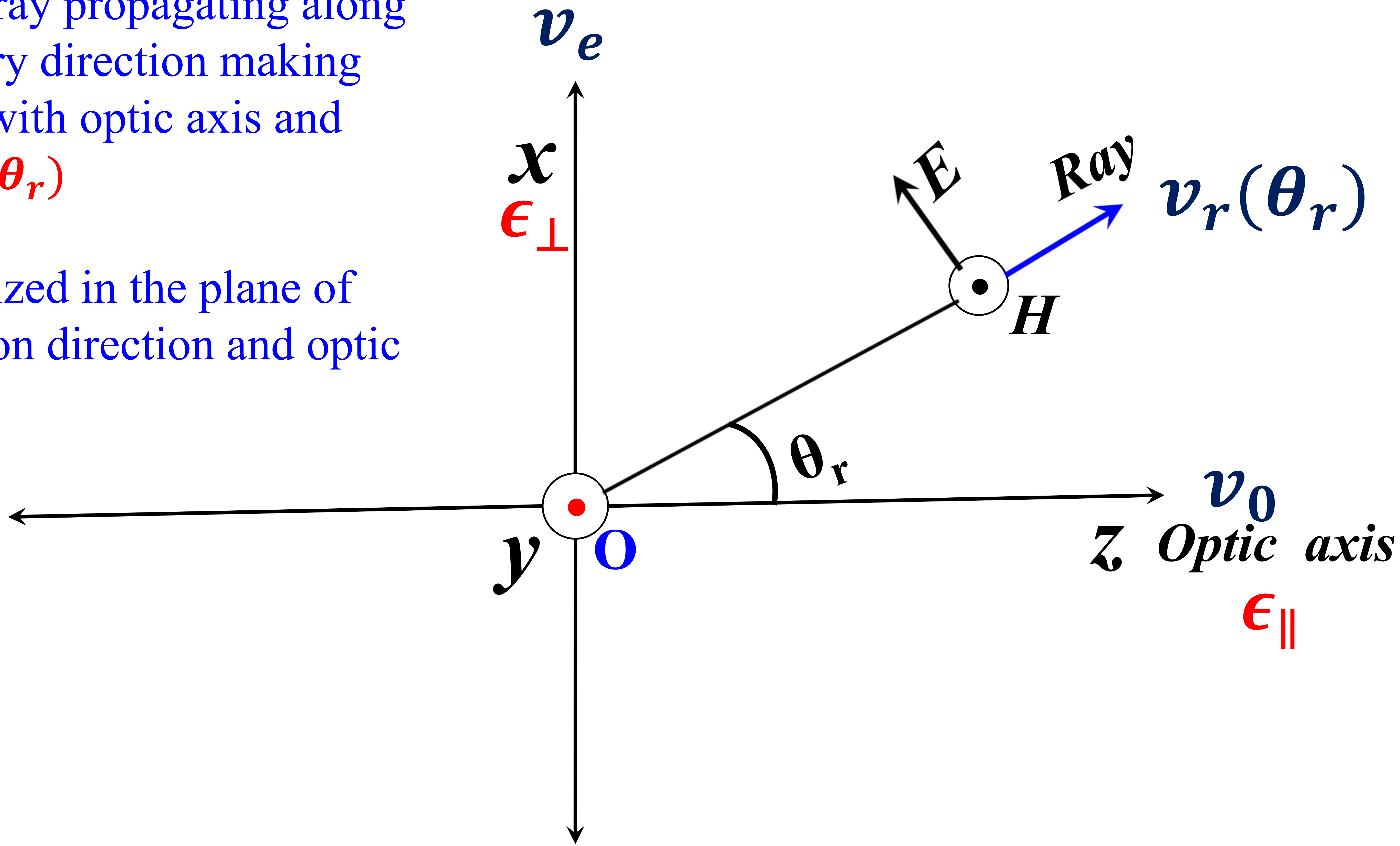


- Consider ray propagating along an arbitrary direction making angle $\boldsymbol{\theta_r}$ with optic axis and speed $\boldsymbol{v_r(\theta_r)}$



- Consider ray propagating along an arbitrary direction making angle $\boldsymbol{\theta_r}$ with optic axis and speed $\boldsymbol{v_r(\theta_r)}$

- It is polarized in the plane of propagation direction and optic axis.



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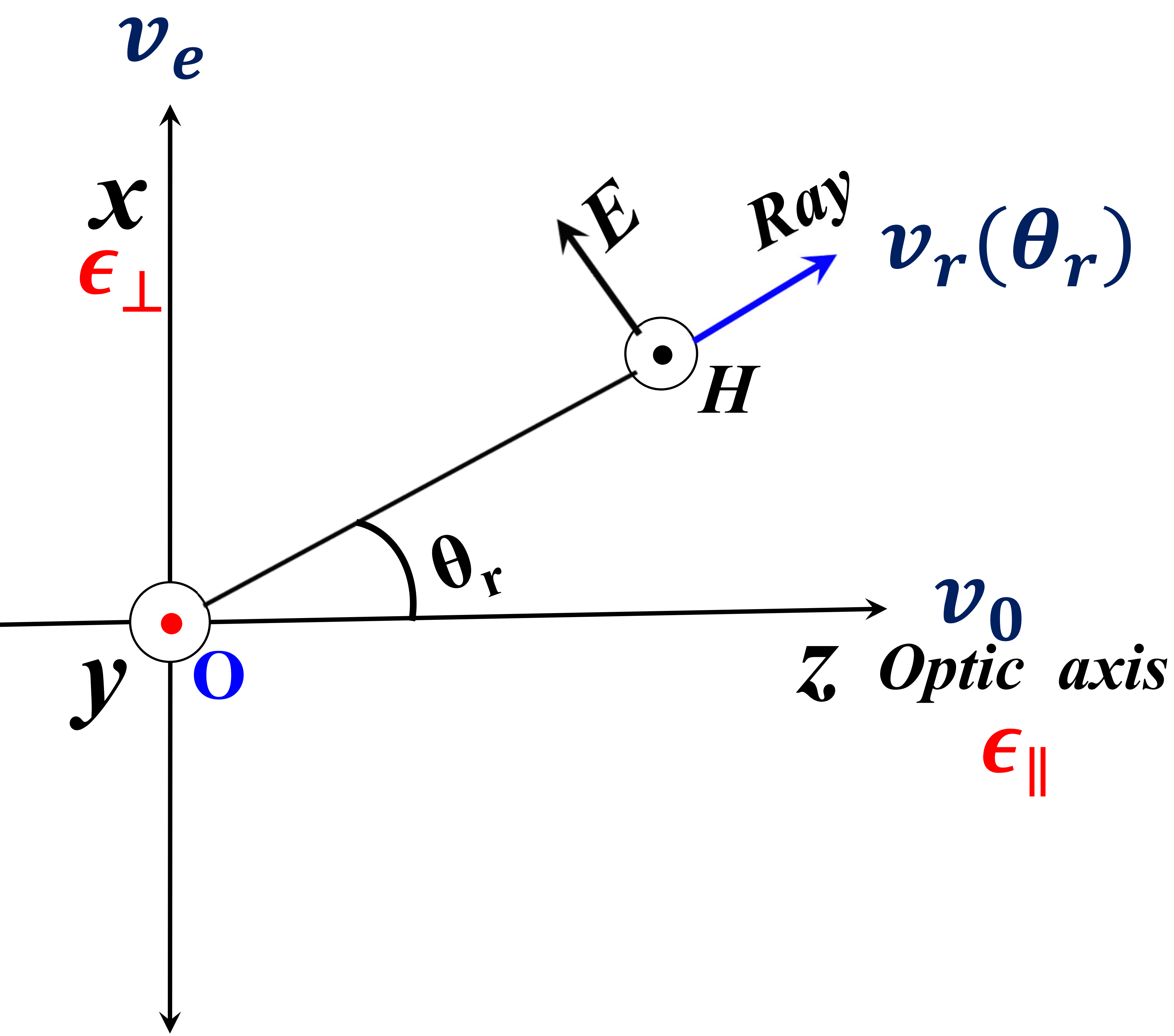
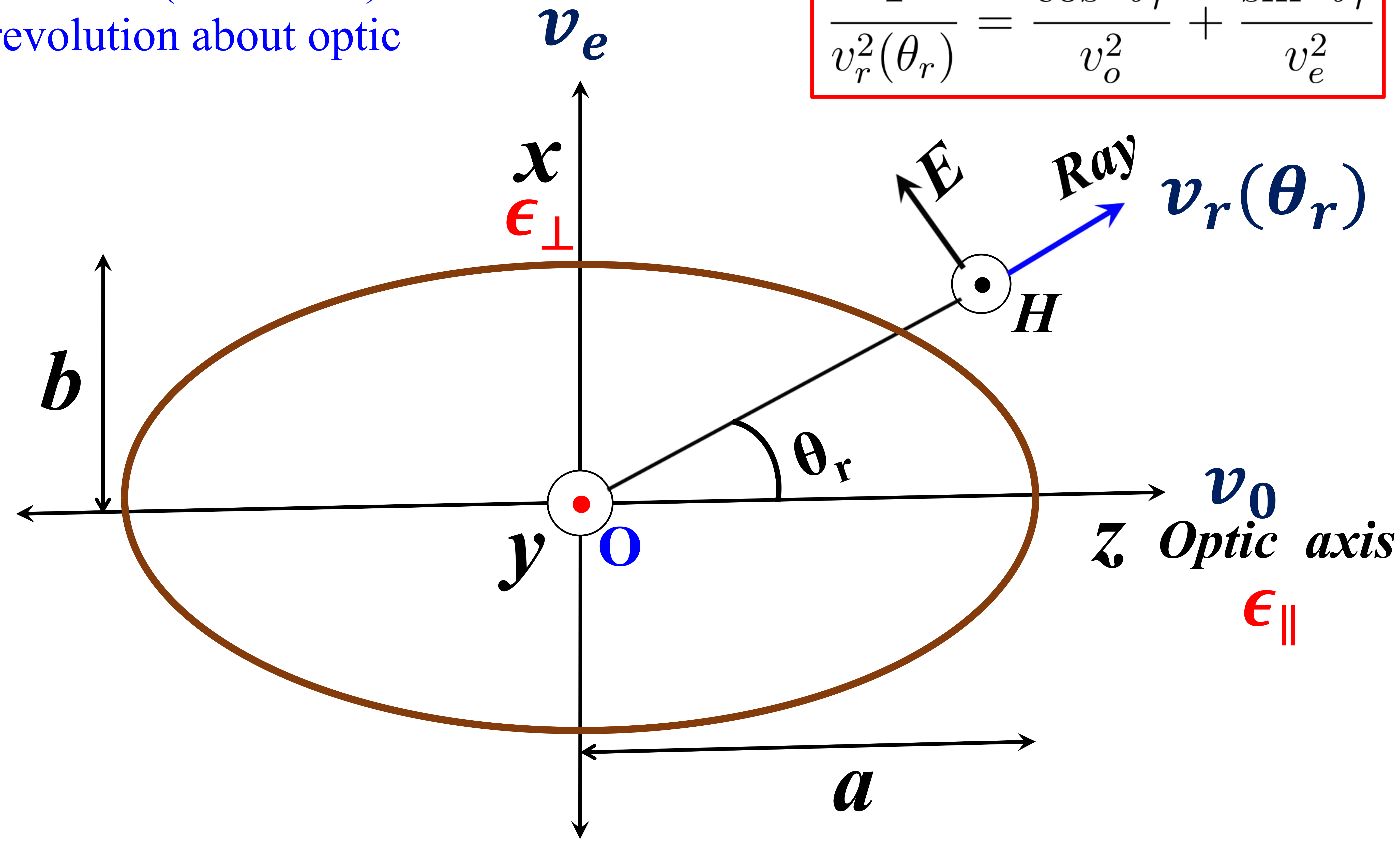
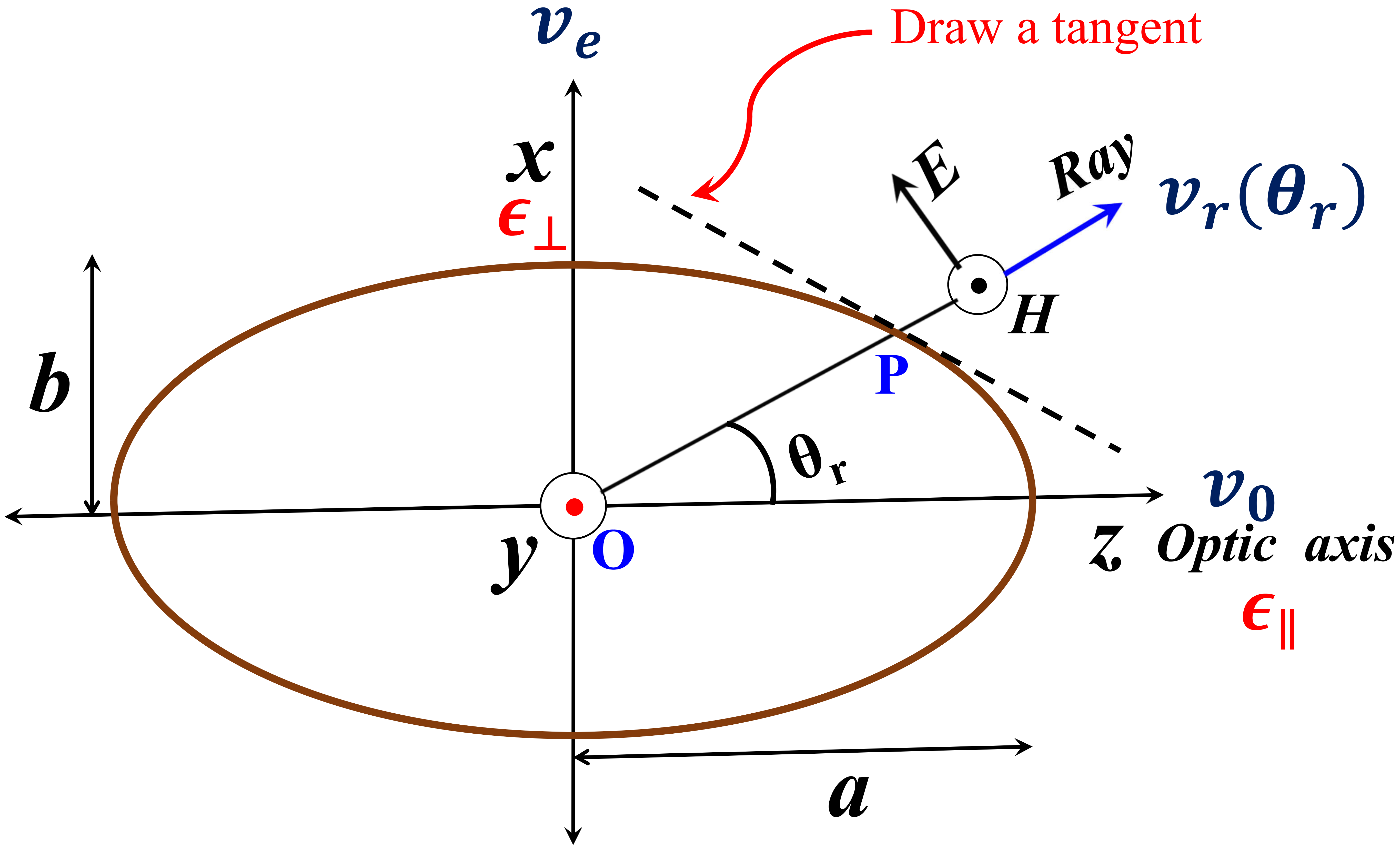


- Since the polarization ($\boldsymbol{E}$) have both x and z component, it experiences both $\epsilon_\parallel$ (z-axis) and $\epsilon_\perp$(x-axis). The ray velocity depends on propagation direction

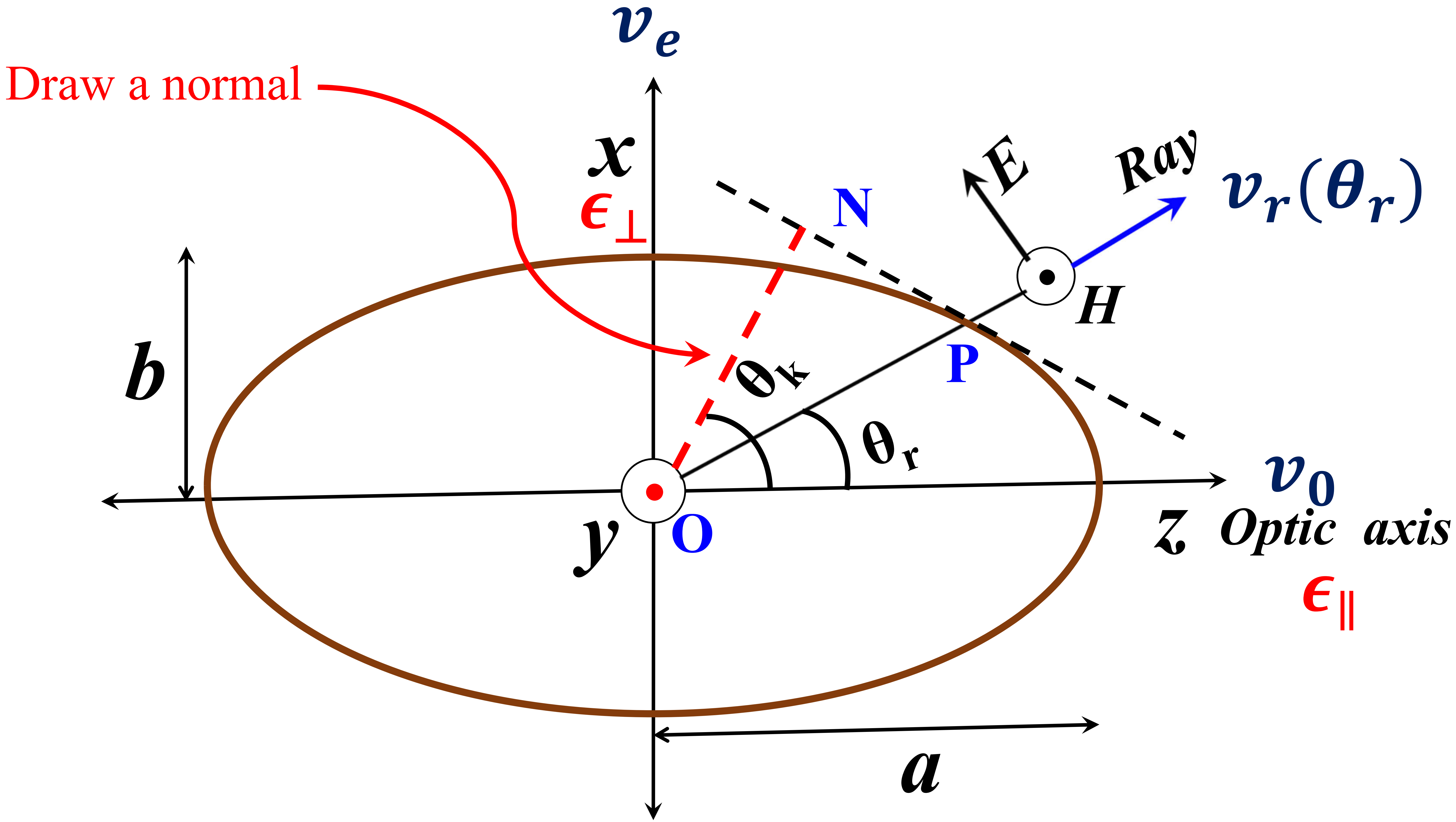
Ray velocity surface (wavefront) is ellipsoid of revolution about optic axis.

$$\frac{1}{v_r^2(\theta_r)} = \frac{\cos^2\theta_r}{v_o^2} + \frac{\sin^2\theta_r}{v_e^2}$$

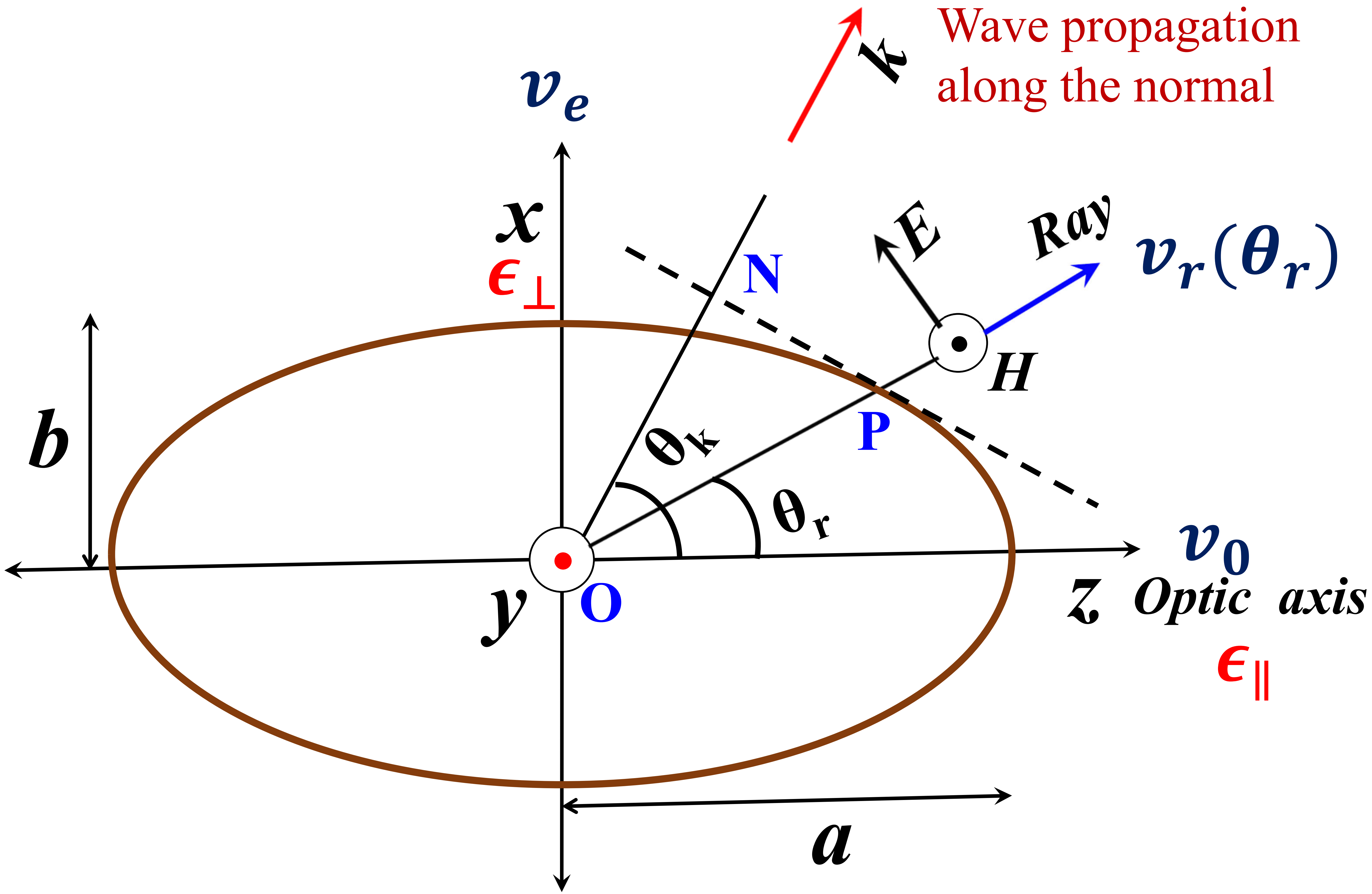




Draw a tangent
v_e
x
$\epsilon_\perp$
E
Ray
$v_r(\theta_r)$
H
P
b
θ_r
v_0
y
O
z
Optic axis
$\epsilon_\parallel$
a



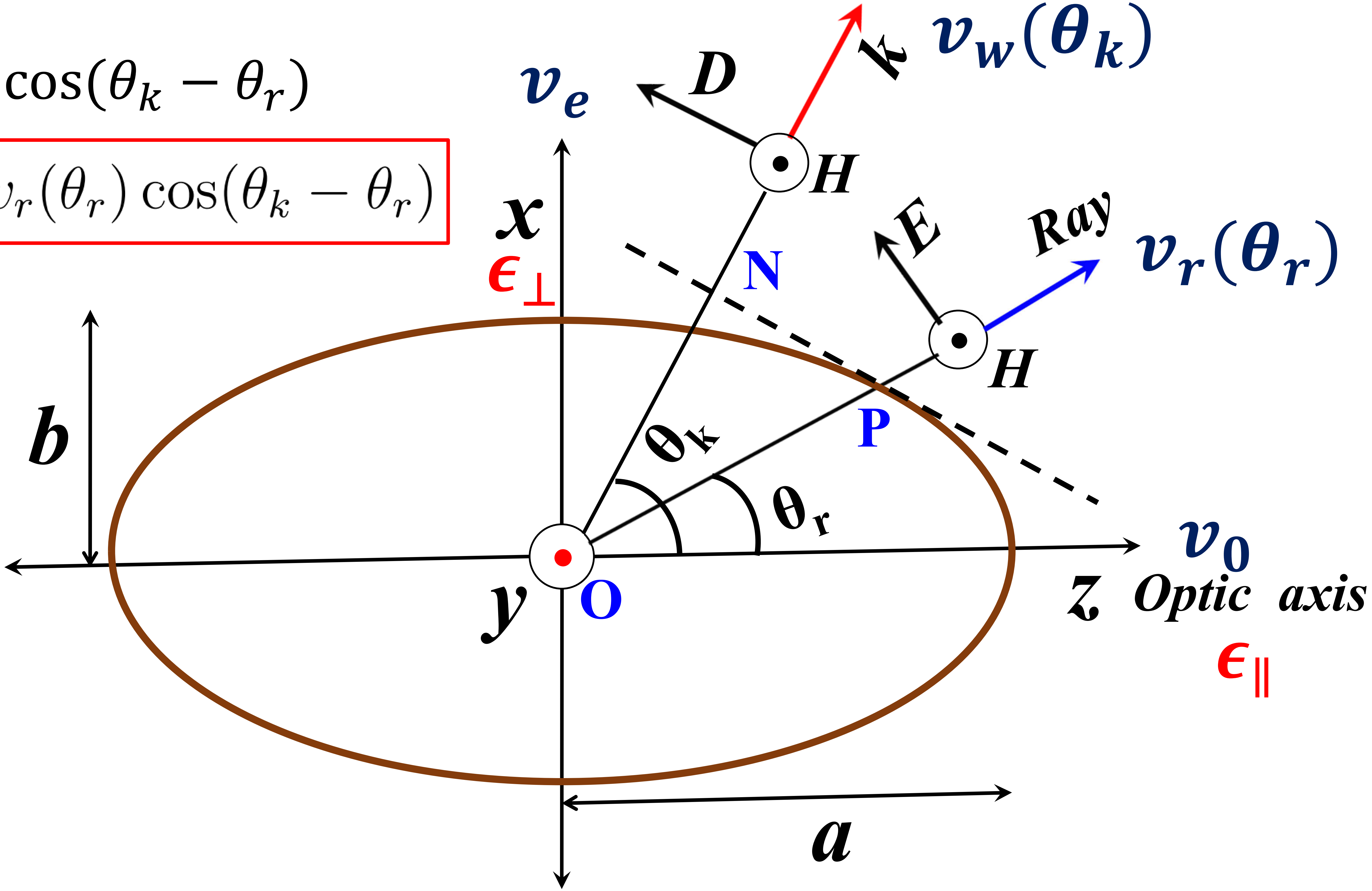
v_e
Draw a normal
x
$\epsilon_\perp$
N
E
Ray
$v_r(\theta_r)$
H
P
θ_k
θ_r
b
v_0
y
O
z Optic axis
$\epsilon_\parallel$
a

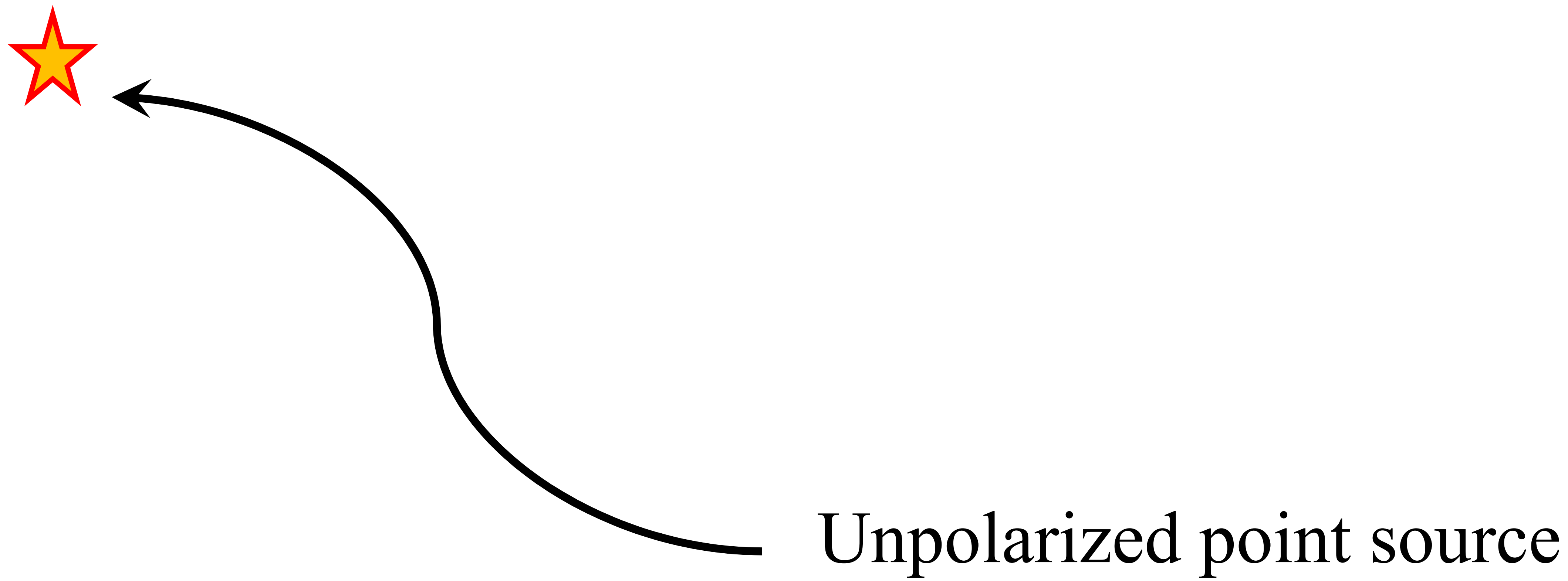


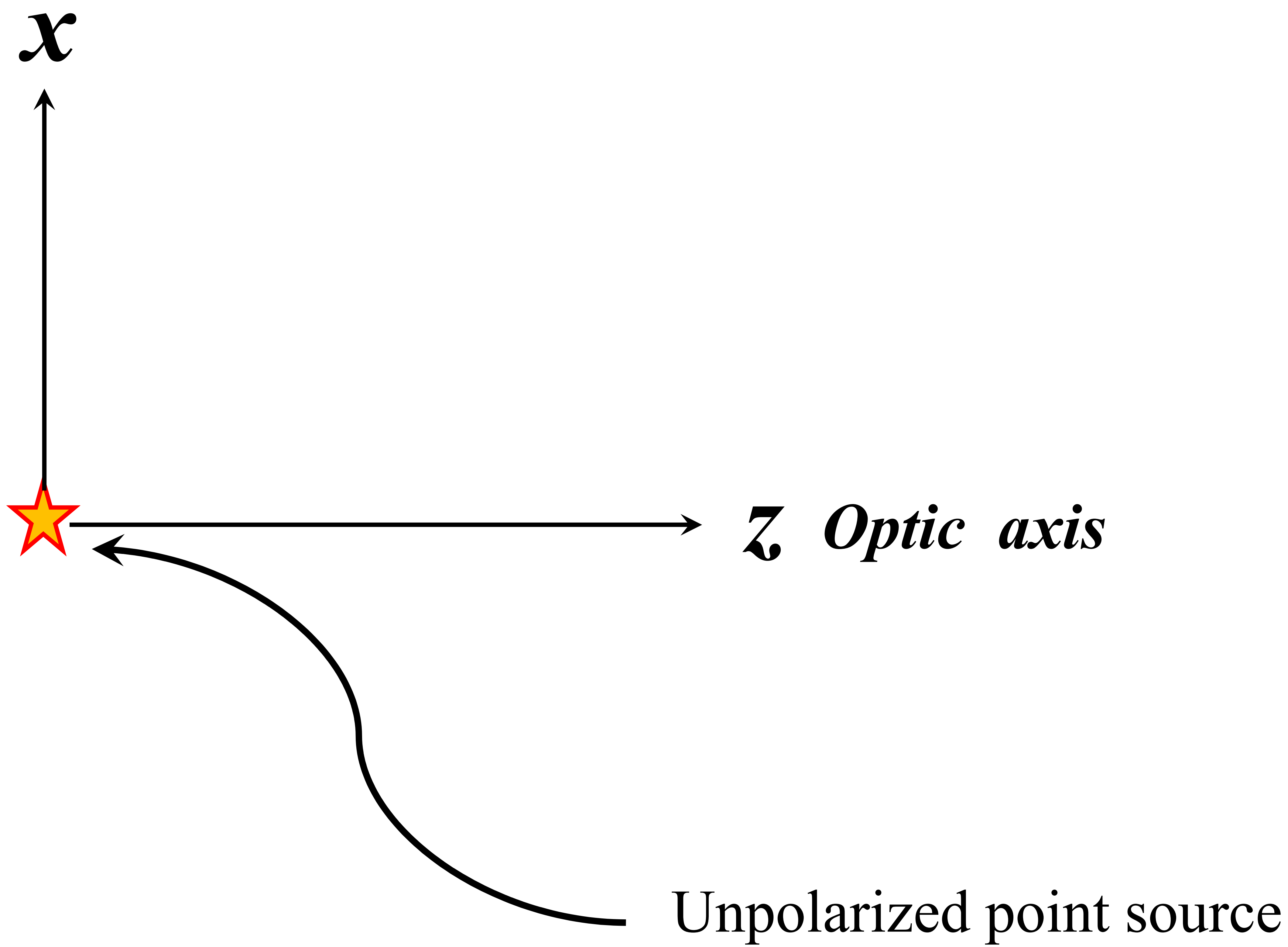
Wave propagation
along the normal
k
v_e
x
$\epsilon_\perp$
N
E
Ray
$v_r(\theta_r)$
H
P
b
θ_k
θ_r
v_0
y
O
z
Optic axis
$\epsilon_\parallel$
a

$$ON = OP\cos(\theta_k - \theta_r)$$

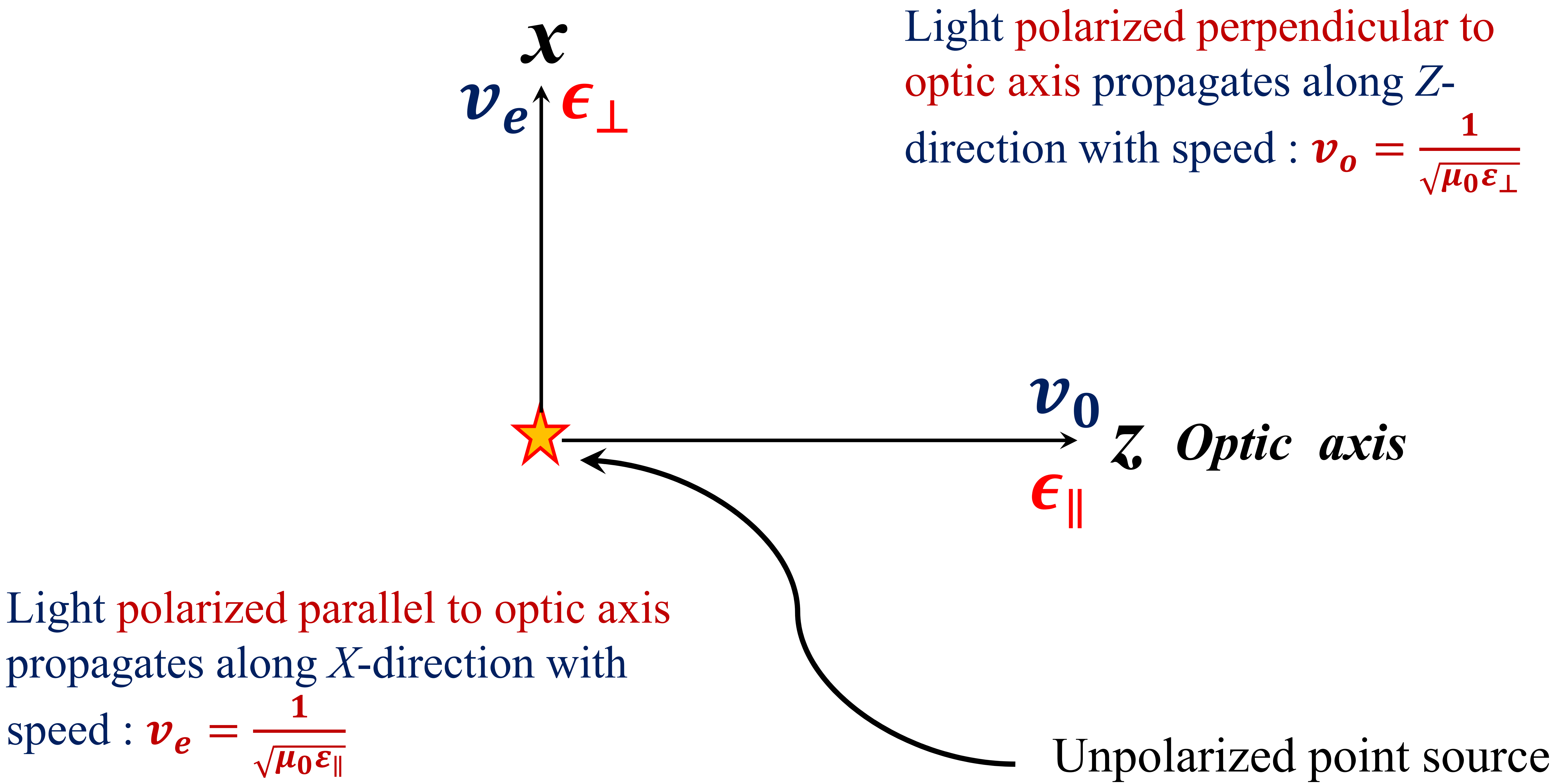
$$v_w(\theta_k) = v_r(\theta_r)\cos(\theta_k - \theta_r)$$



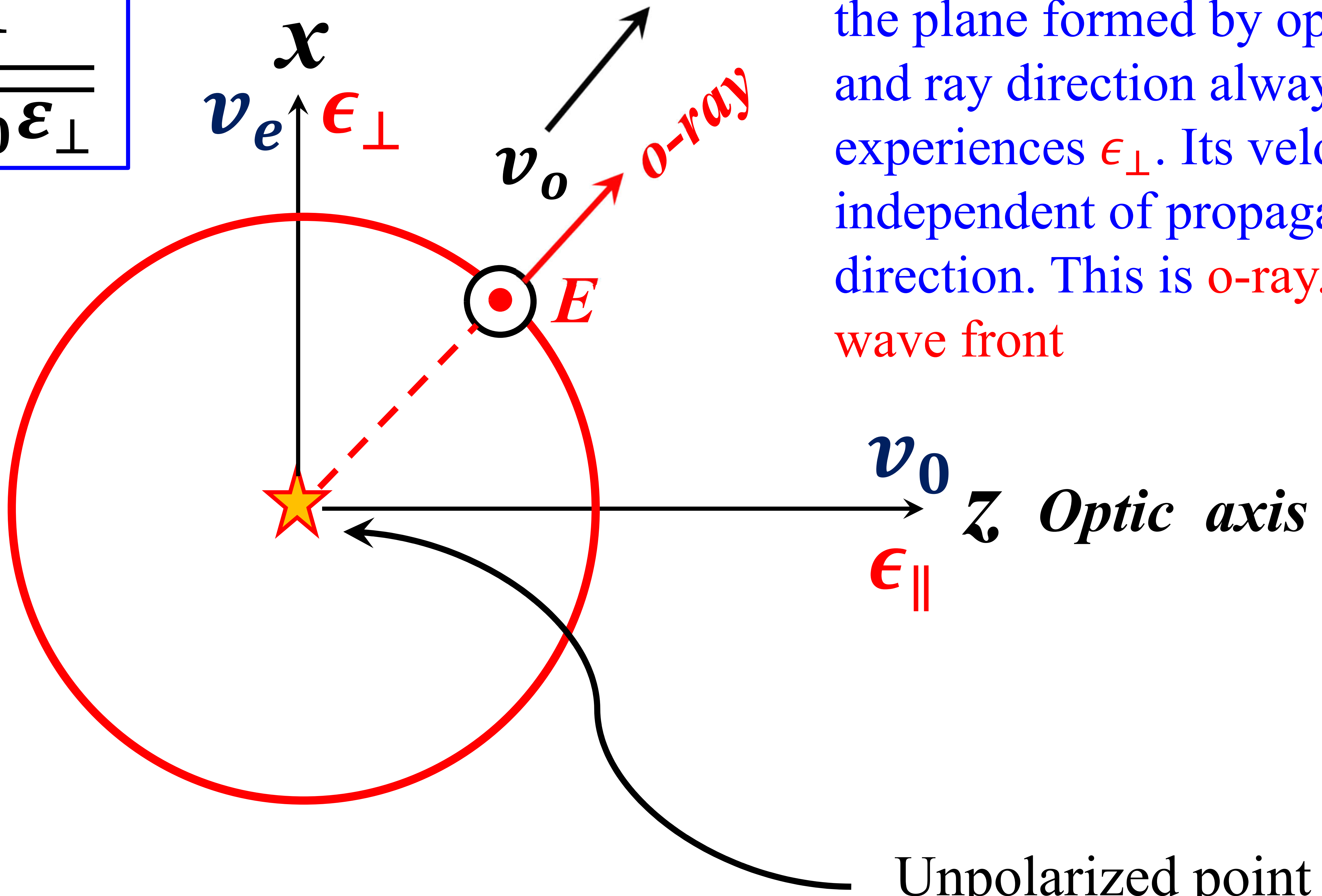

Unpolarized point source



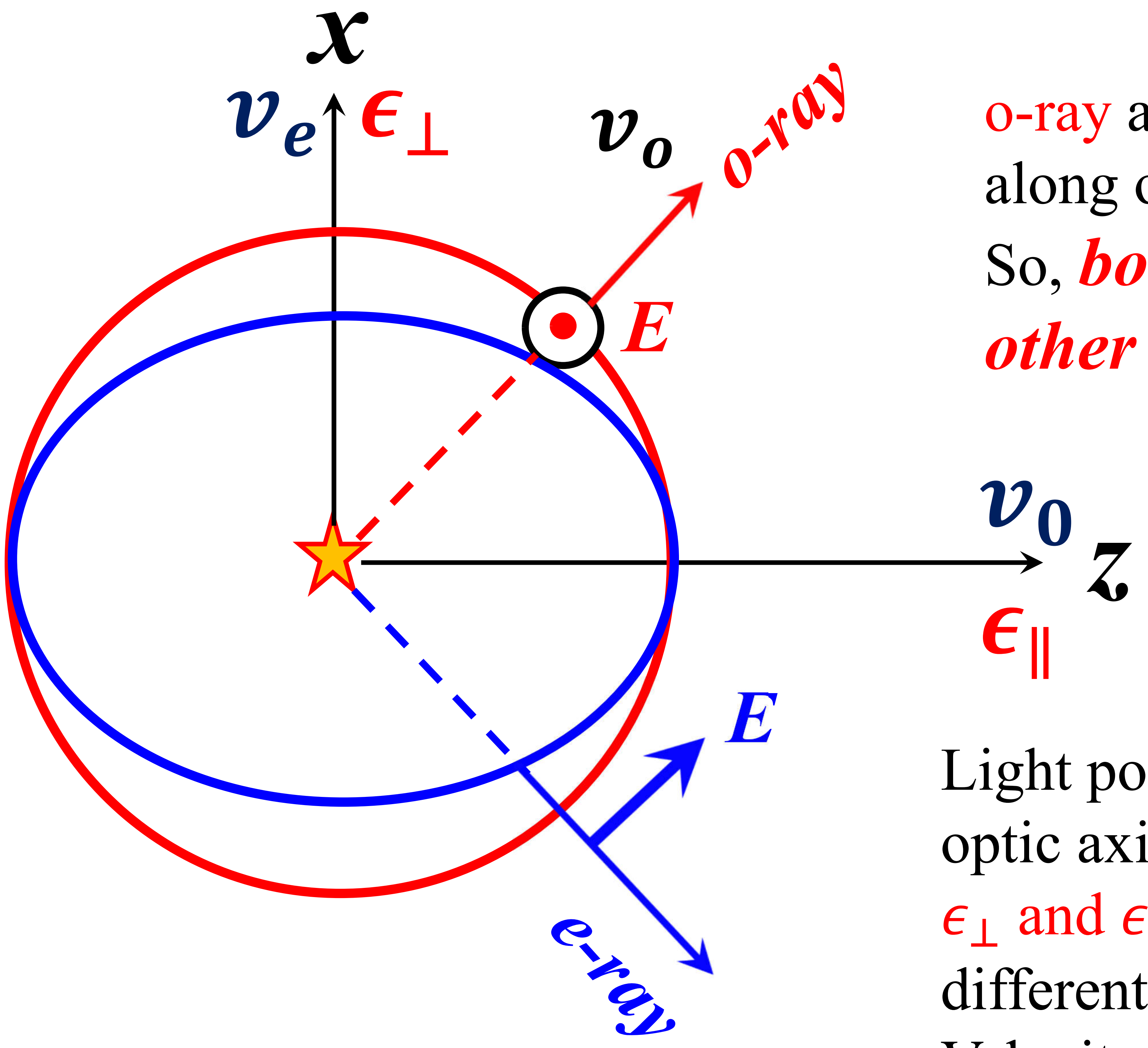
x
z Optic axis
Unpolarized point source


Light polarized perpendicular to optic axis propagates along Z-direction with speed : $v_o = \frac{1}{\sqrt{\mu_0 \varepsilon_\perp}}$
x
v_e
$\epsilon_\perp$
v_0
z Optic axis
$\epsilon_\parallel$
Light polarized parallel to optic axis propagates along X-direction with speed : $v_e = \frac{1}{\sqrt{\mu_0 \varepsilon_\parallel}}$
Unpolarized point source

$$v_o = \frac{1}{\sqrt{\mu_0 \varepsilon_\perp}}$$

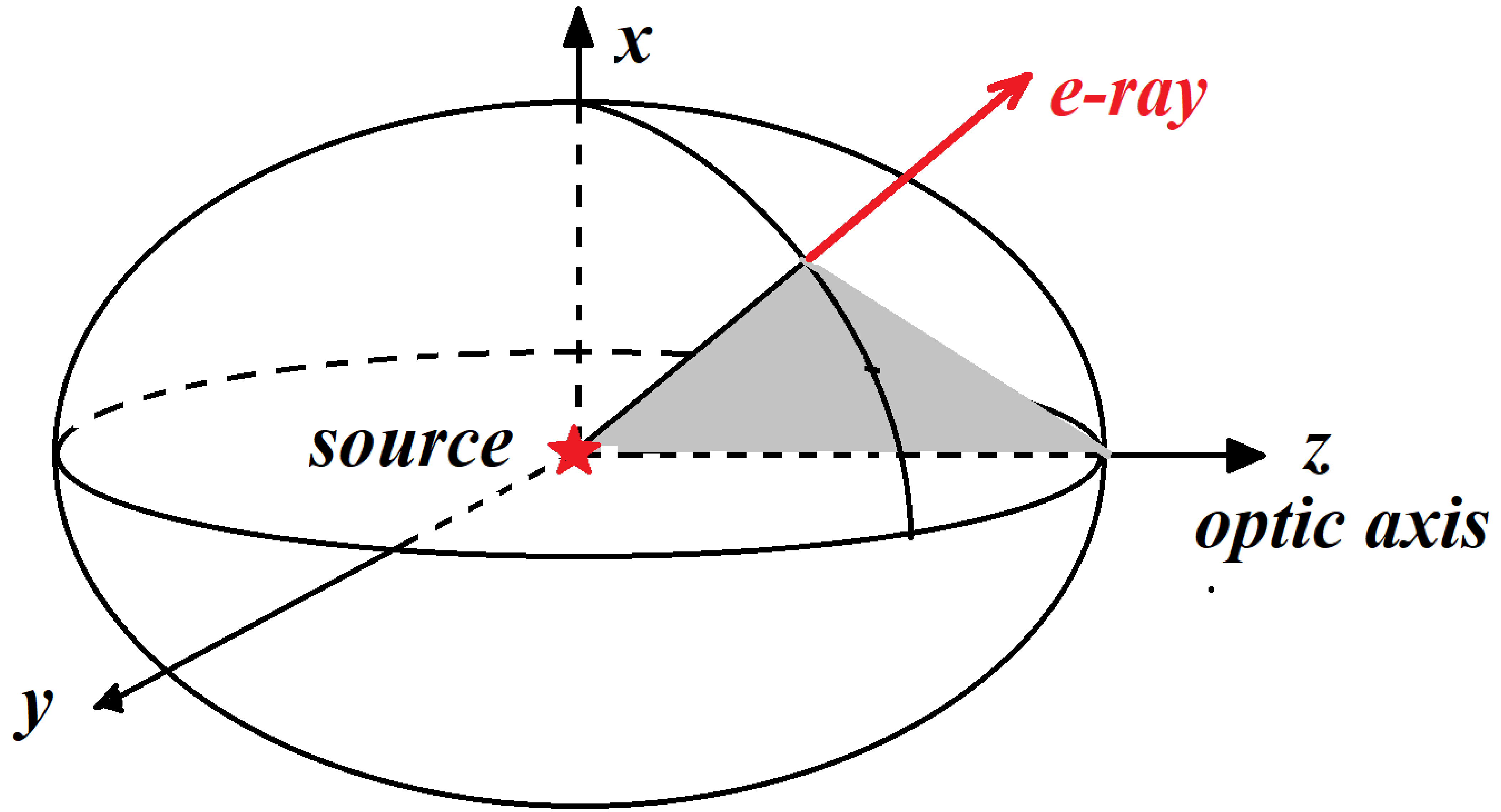


Light polarized perpendicular to the plane formed by optic axis and ray direction always experiences $\epsilon_\perp$. Its velocity is independent of propagation direction. This is o-ray. Spherical wave front



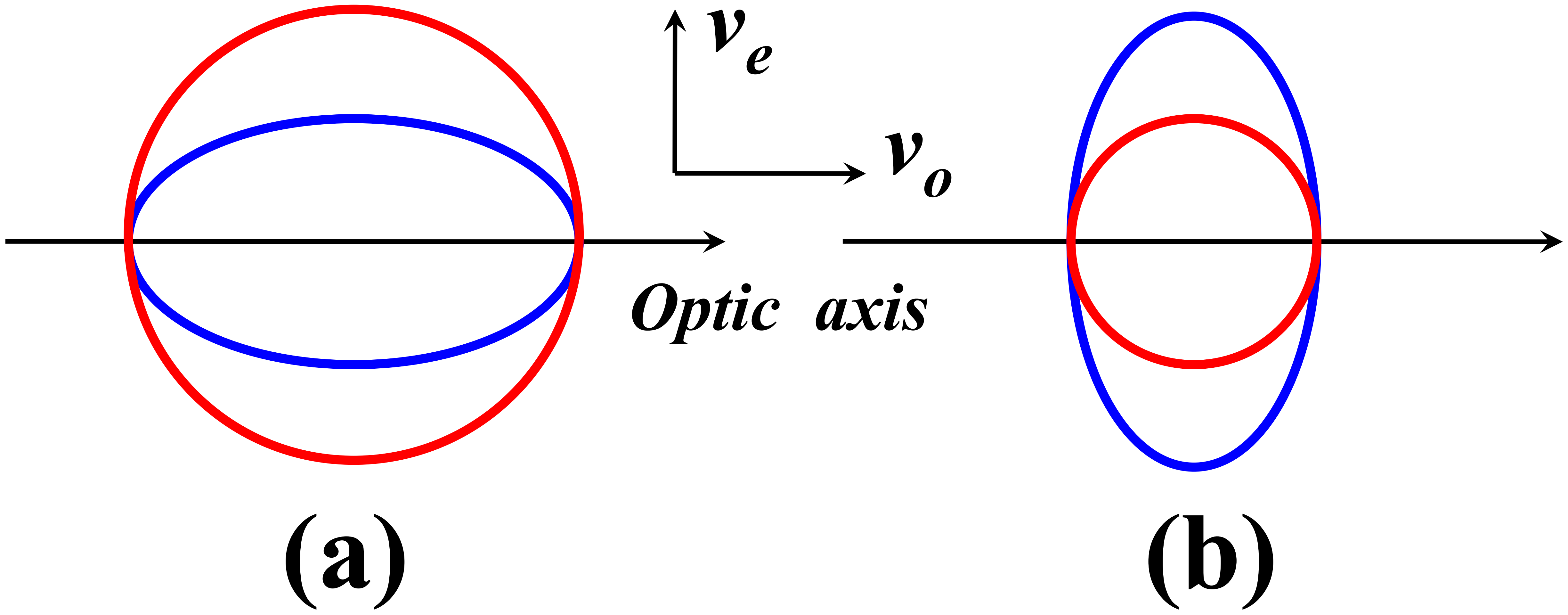
o-ray and e-ray have identical speed along optic axis.
So, ***both wavefront touches each other on optic axis***

Light polarized in the plane formed by optic axis and ray direction experiences $\epsilon_\perp$ and $\epsilon_\parallel$ both. Their influences are different along different direction. Velocity depends on propagation direction. This is e-ray

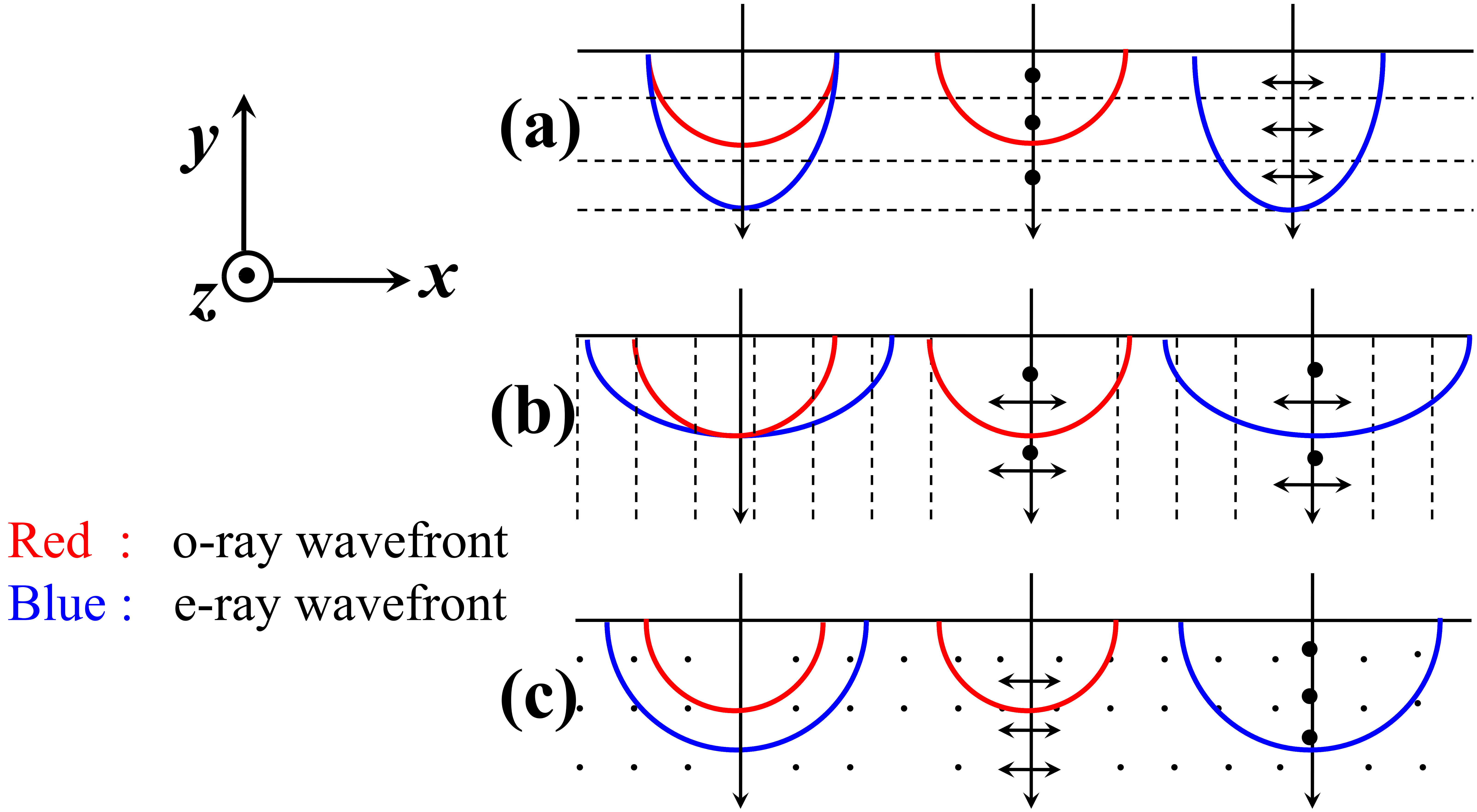


Three dimensional representation of e-ray wavefront.

- It is an ellipsoid of revolution about optic axis.
- Electric field vector lies on the shaded plane formed by ray direction and optic axis.



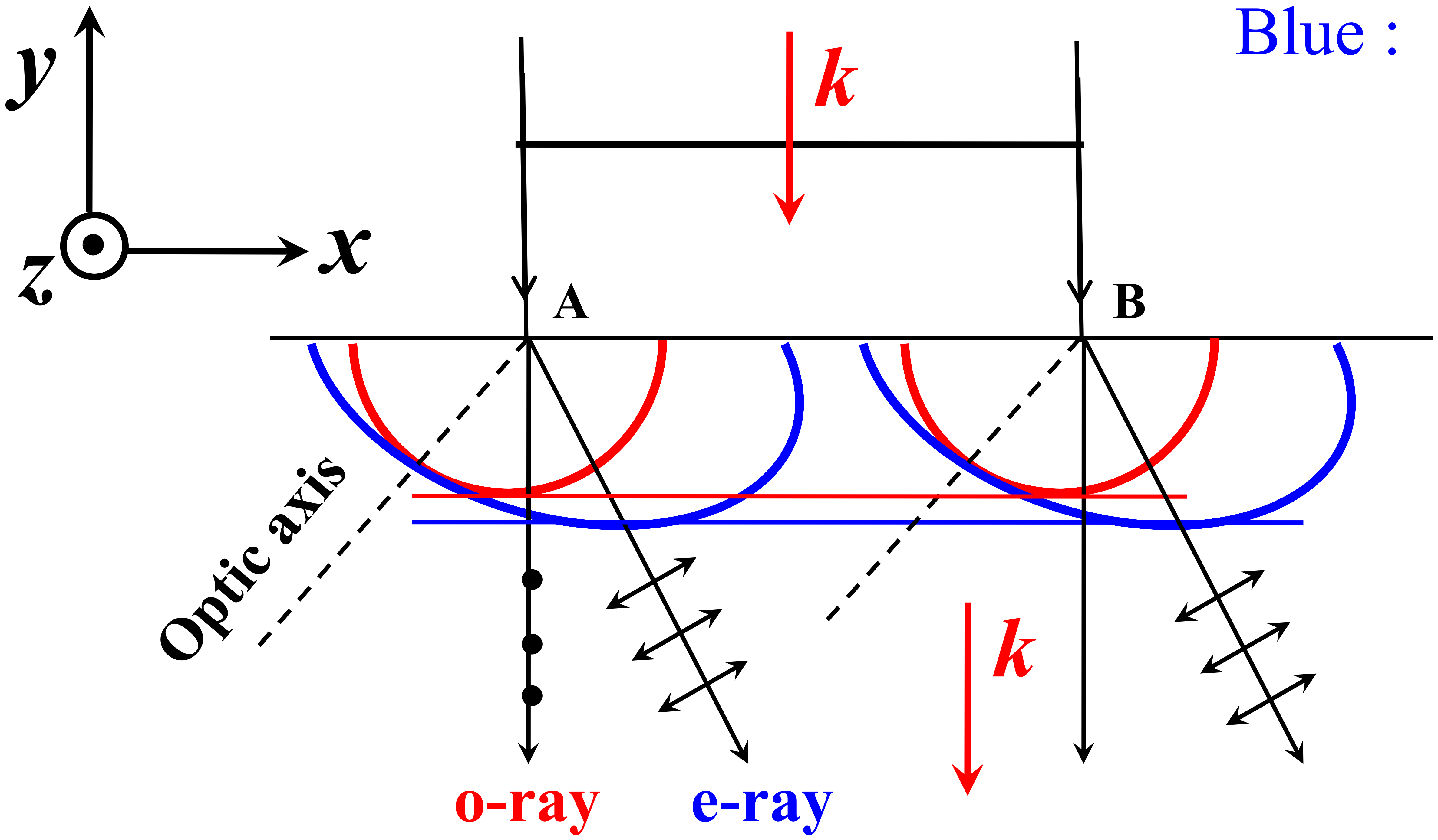
Red : o-ray wavefront
Blue : e-ray wavefront



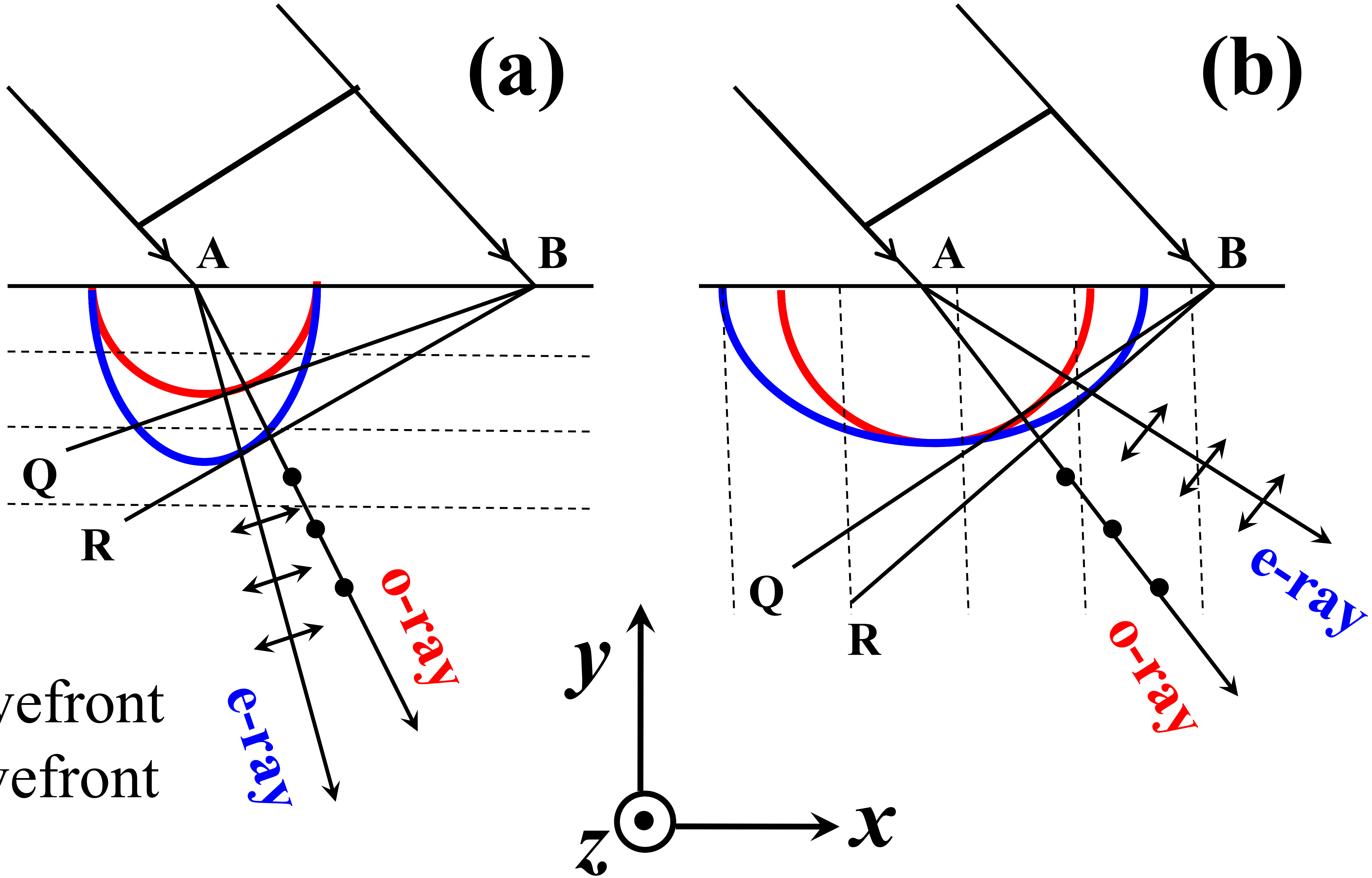
y
x
z
(a)
(b)
(c)
Red : o-ray wavefront
Blue : e-ray wavefront

Normal incidence and optic axis inclined with interface

Red : o-ray wavefront
Blue : e-ray wavefront



Oblique incidence and optic axis (a) parallel and (b) perpendicular to interface



Red : o-ray wavefront
Blue : e-ray wavefront

Oblique incidence and optic axis inclined with interface

Red : o-ray wavefront
Blue : e-ray wavefront