Case I :— $c_j \notin c_B$

$$\Delta c_j \leq z_j - c_j = \Delta_j \quad \text{—— (1)}$$

Case II :— $c_j \in c_B$

$$\underset{x_{rj} > 0}{\underset{j}{\text{Max}}} \left[\frac{-\Delta_j}{x_{rj}} \right] \leq \Delta c_{Br} \leq \underset{x_{rj} < 0}{\underset{j}{\text{Min}}} \left[\frac{-\Delta_j}{x_{rj}} \right] \quad \text{—— (2)}$$

The value of objective f^n may change if c_{Br} is changed to $c_{Br} + \Delta c_{Br}$

$$z = c_B x_B = \sum_{i=1}^{m} c_{Bi} x_{Bi}$$

$$c_{Br} \longrightarrow c_{Br} + \Delta c_{Br}$$

$$\hat{z} = \sum_{i=1, i \neq r}^{m} c_{Bi} x_{Bi} + (c_{Br} + \Delta c_{Br}) x_{Br}$$

$$= \left(\sum_{i=1, i \neq r}^{m} c_{Bi} x_{Bi} + c_{Br} x_{Br} \right) + \Delta c_{Br} x_{Br}$$

$$= \sum_{i=1}^{m} c_{Bi} x_{Bi} + \Delta c_{Br} x_{Br}$$

$$\hat{z} = z + \Delta c_{Br} x_{Br} \quad \text{—— (3)}$$

Ex :— The following table gives the optimal sol^n of a L.P.P.

		$c_j \rightarrow$	3	5	4	0	0	0
Basic Var	c_B	x_B	x_1	x_2	x_3	x_4	x_5	x_6
x_2	✓5	✓50/41	0	1	0	15/41	8/41	−10/41
x_3	✓4	✓62/41	0	0	1	−6/41	5/41	4/41
x_1	✓3	✓89/41	1	0	0	−2/41	−12/41	15/41
[illegible]		765	0	0	0	45/	24/	11/

$z = c_B x_B = \frac{?00}{41}$ | $\Delta_j \rightarrow$ $/41$ $/41$ $/41$

How much c_3 and c_4 can be increased before the present basic solⁿ will no longer be optimal. Also find the change in the value of the obj. f^n.

$\Rightarrow$ Case I:—

$$c_4 \notin c_B = [5, 4, 3] = [c_2, c_3, c_1]$$

$$\Delta c_4 \leq z_4 - c_4 = \Delta_4$$

$$\Rightarrow \Delta c_4 \leq 45/41$$

$\therefore$ the range of c_4 to maintain the optimality of the solⁿ,

$$-\infty < c_4 \leq c_4 + \Delta c_4$$

$$\Rightarrow -\infty < c_4 \leq 0 + 45/41$$

$$\Rightarrow -\infty < c_4 \leq 45/41$$

Case II:—

$$c_3 \in c_B$$

$$(c_{B1}, c_{B2}, c_{B3}) = (c_2, c_3, c_1) = (5, 4, 3)$$

$$c_{B2} \equiv c_3$$

$$c_{B2} \longrightarrow c_{B2} + \Delta c_{B2}$$

From (2),

$$\underset{\substack{j \\ x_{2j} > 0}}{\text{Max}} \left[\frac{-\Delta_j}{x_{2j}}\right] \leq \Delta c_{B2} \leq \underset{\substack{j \\ x_{2j} < 0}}{\text{Min}} \left[\frac{-\Delta_j}{x_{2j}}\right]$$

$$\underset{\substack{j \\ x_{2j} > 0}}{\text{Max}} \left[\frac{-\Delta_3}{x_{23}}, \frac{-\Delta_5}{x_{25}}, \frac{-\Delta_6}{x_{26}}\right] \leq \Delta c_{B2}$$

$$\le \underset{\substack{j \\ x_{2j}<0}}{\text{Min}} \left[\frac{-c_4}{x_{24}} \right]$$

$$\Rightarrow \underset{\substack{j \\ x_{2j}>0}}{\text{Max}} \left[\frac{-0}{1}, \frac{-24/41}{5/41}, \frac{-11/41}{4/41} \right]$$

$$\le \Delta c_{B2} \le \underset{\substack{j \\ x_{2j}<0}}{\text{Min}} \left[\frac{-45/41}{-6/41} \right]$$

$$\Rightarrow \quad 0 \le \Delta c_{B2} \le 45/6$$

$$\Rightarrow \quad c_{B2} + 0 \le c_{B2} \le c_{B2} + 45/6$$

$$\Rightarrow \quad 4 \le c_{B2} \le 4 + \frac{45}{6}$$

Change in the value of z :⟶

from (3),

$$\hat{z} = z + \Delta c_{B2}\, x_{B2}$$

$$= \frac{765}{41} + \left(0 \le \Delta c_{B2} \le \frac{45}{6}\right) \times \frac{82}{41}$$

$$\Delta c_{B2} = 3$$

$$c_3 \to 4 \to 7$$

$$\frac{765}{41} + 3 \times x_{B2}$$

Solve Max $z = 3x_1 + 5x_2 + 4x_3$

s.t. —— (3)

Max $z = 3x_1 + 5x_2 + 7x_3$

s.t. same

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2) Find the optimal solⁿ to the LPP : $\text{Max } z = 15x_1 + 45x_2$

$$\text{s.t.} \quad x_1 + 16x_2 \le 240$$

$$5x_1 + 2x_2 \le 162$$

$$x_2 \le 50$$

$$x_1, x_2 \ge 0.$$

If $\max z = \sum c_j x_j$, $j = 1, 2$, c_2 is kept fixed 45, determine how much c_1 can be changed without effecting the above sol^n.