Changes in the coeff. (c_j) of the obj. function.

		$c_j \rightarrow$	7	2	9	0	0	0
Basic var	C_B	x_B	x_1	x_2	x_3	x_4	x_5	x_6
x_1	7	b_1	1	0	0			
x_2	2	b_2	0	1	0			
x_3	9	b_3	0	0	1			
$\Delta_j = z_j - c_j \rightarrow$			0	0	0	1	2	5

$\Delta_j \geq 0 \longrightarrow$ $\quad Z = C_B x_B$

$\Delta_5 = 2$, $\Delta c_5 \leq \Delta_5 = 2$

① $c_j \notin C_B$

② $c_j \in C_B$

$c_5 = 0 = 0 + \delta = 3$

$c_6 = 7$

for optimality, $c_5' = c_5 + \Delta c_5 \leq 2$

$c_6' \leq 5$

$\Delta c_5 \leq 2$

$\Delta c_6 \leq 5$

Let us consider a L.P.P.

$$\text{Max } Z = CX$$

$$\text{S.t., } AX = b, \; X \geq 0. \longrightarrow (1)$$

Let X_B is the optimal basic feasible solution and B is an optimal basis matrix. Let our problem is non-degenerate. So we have B.F.S to (1).

$$X_B = B^{-1} b$$

If we change some component c_j of C, then X_B remains unaltered. (X_B is independent on C)

$\therefore$ x_B will remain B.F.S.

optimality condition $\Delta_j = z_j - c_j \geq 0$ may not necessarily satisfied when c_j is changed to $c_j + \Delta c_j$.

We have two possible cases —

i) c_j of c is changed where c_j is the coeff. of non-basic variable of the obj. f^n. i.e. $c_j \notin c_B$.

ii) c_j of c is changed where c_j is the coeff. of basic variable of the obj. f^n i.e. $c_j \in c_B$.

Case 1 : —

If $c_j \notin c_B$.

Let c_j is changed to $c_j + \Delta c_j$

We have $\Delta_j = z_j - c_j \geq 0, \forall j$

To preserve the optimality, Δc_j must satisfy the optimality condition

$$\Delta_j' = z_j - (c_j + \Delta c_j) \geq 0.$$

$z_j = c_B x_j$ does not depend on c_j.

$$\Delta c_j \leq z_j - c_j$$

$$\Rightarrow \Delta c_j \leq \Delta_j$$

If, $c_j \notin c_B$ Then we can replace c_j by $c_j + \Delta c_j$ ...

$c_j + \Delta c_j$ such that $\Delta c_j \leq \Delta_j$

Case II :— If $c_j \in c_B$ (c_1, c_2, c_3) → $c_{B1}\ c_{B2}\ c_{B3}$

Let $c_j = c_{Br}$

$$z_j = c_B x_j = \sum_{i=1}^{m} c_{Bi} x_{ij}$$

If we change c_{Br} to $c_{Br} + \Delta c_{Br}$,

Then

$$\hat{z}_j = \sum_{i=1,\, i\neq r}^{m} c_{Bi} x_{ij} + (c_{Br} + \Delta c_{Br}) \cdot x_{rj}$$

$$\hat{\Delta}_j = \hat{z}_j - c_j = \left[\sum_{i=1,\, i\neq r}^{m} c_{Bi} x_{ij} + (c_{Br} + \Delta c_{Br}) x_{rj} \right] - c_j$$

$$= \left[\sum_{i=1}^{m} c_{Bi} x_{ij} + \Delta c_{Br} \cdot x_{rj} \right] - c_j$$

$$= (z_j + x_{rj} \Delta c_{Br}) - c_j$$

$$= (z_j - c_j) + x_{rj} \Delta c_{Br}$$

To preserve the optimality condition, Δc_{Br} must satisfy the optimality condition $\hat{\Delta}_j = \hat{z}_j - c_j \geq 0$

$$\Rightarrow (z_j - c_j) + x_{rj} \Delta c_{Br} \geq 0$$

Now $z_j - c_j \geq 0$ and

$$(z_j - c_j) + x_{rj} \underline{\Delta c_{Br}} \geq 0$$

$$\left[\quad d_j \geq 0, \quad d_j + \lambda d_j' \geq 0 \right.$$

$$\max_{\substack{j \\ (d_j' > 0)}} \left[\frac{-d_j}{d_j'} \right] \leq \lambda \leq \min_{\substack{j \\ (d_j' < 0)}} \left[\frac{-d_j}{d_j'} \right]$$

$$d_j \to z_j - c_j = \Delta_j, \quad d_j' \to x_{rj}, \quad \lambda \to \Delta c_{Br}$$

$$\max_{\substack{j \\ x_{rj} > 0}} \left[\frac{-\Delta_j}{x_{rj}} \right] \leq \Delta c_{Br} \leq \min_{\substack{j \\ x_{rj} < 0}} \left[\frac{-\Delta_j}{x_{rj}} \right]$$