$$\text{Max } Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n \Big\} \text{objective } f^n$$

$$\text{constraints}\left\{\begin{array}{l} \text{S.t.,} \\ a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \\ x_i \geq 0, \ \forall i \end{array}\right.$$

$$\text{Max } Z = c^t x$$

$$\text{S.t. } Ax \leq b, \quad x \geq 0.$$

$$c = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}, \quad b = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}$$

$$A = (a_{ij})_{m \times n}$$

$$\text{Max } Z = cx \qquad c = (c_1, c_2, \ldots, c_n)$$

$$\text{S.t. } Ax \leq b,$$
$$x \geq 0.$$

The investigation that deals with changes in the optimal solution due to changes in the parameters (a_{ij}, b_i, c_j) is called sensitivity / post-optimality analysis.

Changes might happen:—

1) Coeff. (c_j) of the objective f^n:—

a) Coeff. of basic variables $(c_j \in c_B)$

b) coeff. of nonbasic " ($c_j \notin c_B$)

2) Changes in b_i (right hand side constants)

3) Changes in a_{ij} :—

a) coeff. of basic matrix ($a_{ij} \in B$)

b) coeff. of non-basic " ($a_{ij} \notin B$)

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x_B	c_B	c_j . . .			
x_2		a_{11}	a_{12}	:	.
x_1		.	.	:	1
x_4		.	.	:	.

			c_1	c_2	c_3	$c_4=0$	$c_5=0$	$c_6=0$
x_2	x_4	c_4	5	-7	9	1	0	0
x_3	x_5	c_5	0	0	1	0	1	0
x_1	x_6	c_6		0	0	0	0	1

c_B Max $z = c_1x_1 + c_2x_2 + c_3x_3 + 0.x_4 + 0.x_5 + 0.x_6$

$5x_1 + 7x_2 + 9x_3 \leq 2$

$. \quad . \quad . + x_5 \leq -3 \qquad 0.x_4 + 1.x_5 + 0.x_6$

$. \quad . \quad . + x_6 \leq 8 \qquad 0.x_4 + 0.x_5 + 1.x_6$

	c_B	x_B	b	c_1	c_2	c_3	0	0	0	
	0	x_4	2	5	7 0	9	1	-9	0	
x_2	c_2	x_5	-3	-1	$\boxed{2}$ 1	5	0	11	0	$\longrightarrow x_5$
	0	x_6	8		0		0	8	1	
	$z_j - c_j$			✓	✓	✓	.	.	.	

4) Addition of new variables to the problem

5) Addition of new constraints.

Lemma :—

If $d_j \geq 0$, $d_j + \lambda d_j' \geq 0$, $j = 1(1)n$,

then, $$\max_{\substack{j \\ (d_j' > 0)}} \left[-\frac{d_j}{d_j'} \right] \leq \lambda \leq \min_{\substack{j \\ (d_j' < 0)}} \left[-\frac{d_j}{d_j'} \right]$$

$\Rightarrow$ Since $d_j \geq 0$ and $d_j + \lambda d_j' \geq 0$, $j = 1(1)n$.

we have three possibilities:

a) For those j for which $d_j' > 0$,

$$\lambda \geq -\frac{d_j}{d_j'}$$

$$\Rightarrow \lambda \geq \max_{\substack{j \\ d_j' > 0}} \left[-\frac{d_j}{d_j'} \right],$$

b) For those j for which $d_j' < 0$,

$$\lambda d_j' \geq -d_j$$

$$\Rightarrow \lambda \leq \left[-\frac{d_j}{d_j'} \right]$$

$$\Rightarrow \quad \lambda \leq \min_{\substack{j \\ d_j' < 0}} \left[-\frac{d_j}{d_j'} \right]$$

c) for those j, for which $d_j' = 0$.

We have λ as unrestricted.

So combining (a), (b), (c), we have,

$$\max_{\substack{j \\ d_j' > 0}} \left[-\frac{d_j}{d_j'} \right] \leq \lambda \leq \min_{\substack{j \\ d_j' < 0}} \left[-\frac{d_j}{d_j'} \right]$$

If $d_j' \geq 0$, $\forall j$, then λ has no upper bound.

If $d_j' \leq 0$, $\forall j$, then λ has no lower bound.