Ex:

$$\text{Max } Z = 3x_1 + 2x_2 + 5x_3$$

$$\text{s.t.}, \quad x_1 + 2x_2 + x_3 \le 430 + 100\theta$$

$$3x_1 + 2x_3 \le 460 - 200\theta$$

$$x_1 + 4x_2 \le 420 + 400\theta$$

$$x_1, x_2, x_3 \ge 0.$$

Determine the range of θ for which the solution remains optimal basic feasible.

$\Rightarrow$ for $\theta = 0$,

Optimal table

		$c_j \rightarrow$	3	2	5	0	0	0	
Basic var	c_B	x_B	x_1	x_2	x_3	x_4	x_5	x_6	
x_2	2	100	$-1/4$	1	0	$1/2$	$-1/4$	0	
x_3	5	230	$3/2$	0	1	0	$1/2$	0	
x_6	0	20	2	0	0	-2	1	1	
	$Z = 1350$		4	0	0	1	2	0	$\leftarrow \Delta_j$

$$x_B = \begin{pmatrix} 100 \\ 230 \\ 20 \end{pmatrix}$$

for $\theta > 0$, $$b^{\theta} = \begin{pmatrix} 430 \\ 460 \\ 420 \end{pmatrix} + \theta \begin{pmatrix} 100 \\ -200 \\ 400 \end{pmatrix}$$

$$= b + \theta g$$

$$g = \begin{pmatrix} 100 \\ -200 \\ 400 \end{pmatrix}$$

$$P = B^{-1} g = \begin{pmatrix} 1/2 & -2/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 100 \\ -200 \\ 400 \end{pmatrix}$$

$$= \begin{pmatrix} 100 \\ -100 \\ 0 \end{pmatrix}$$

$p_1 = 100, \quad p_2 = -100; \quad p_3 = 0.$

$$\beta = \alpha + \min_{p_i < 0} \left(\frac{-x_{B_i}^{\alpha}}{p_i} \right)$$

Let next critical value θ'.

$$\theta' = 0 + \left(\frac{-x_{B2}^{0}}{p_2} \right)$$

$$= 0 + \frac{-230}{-100}$$

$$= 2.3$$

$\therefore$ first critical value after $\theta = 0$ is $\theta' = 2.3$.

Solution remains Basic & feasible for $0 \le \theta < 2.3$.

At $\theta' = 2.3$,

$$b^{\theta'} = \begin{pmatrix} 430 \\ 460 \\ 420 \end{pmatrix} + 2.3 \begin{pmatrix} 100 \\ -200 \\ 400 \end{pmatrix}$$

$$b + \theta' g$$

$$= \begin{pmatrix} 660 \\ 0 \\ 1340 \end{pmatrix}$$

Corresponding basic solution,

$$x_B^{\theta'} = B^{-1} b^{\theta'} = \begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 660 \\ 0 \\ 1340 \end{pmatrix}$$

$$= \begin{pmatrix} 330 \\ 0 \\ 20 \end{pmatrix}$$