Linear Variations in b-vector :—

$$\text{Max } Z = CX$$

$$\text{s.t. } AX = b, \quad X \geq 0$$

$$b^{\theta} = b + \theta g$$

$$\Delta_j^{\theta} = Z_j^{\theta} - c_j^{\theta} \geq 0,$$ optimality condition will remain satisfied.

$$X_B = B^{-1} b \geq 0$$

check feasibility of the solution

$\theta = 0$

$\alpha \leq \beta$, α, β critical values

$$X_B^{\alpha} = B_{\alpha}^{-1} b^{\theta} \geq 0$$

feasible for some range of $\theta \geq \alpha$.

$$X_B^{\theta} = B_{\alpha}^{-1} b^{\theta}$$

$$= B_{\alpha}^{-1} \left[\underset{b}{\overset{\shortparallel}{b^0}} + \theta g \right]$$

$$= B_{\alpha}^{-1} \left[b^0 + \alpha g + (\theta - \alpha) g \right]$$

$$= B_{\alpha}^{-1} \left(b^0 + \alpha g \right) + (\theta - \alpha) B_{\alpha}^{-1} g$$

$$= B_{\alpha}^{-1} \left(b^{\alpha} \right) + (\theta - \alpha) B_{\alpha}^{-1} g$$

$$X_B^{\theta} = X_B^{\alpha} + (\theta - \alpha) P, \quad P = B_{\alpha}^{-1} g$$

X_B^{θ} will remain feasible if

$$X_B^{\theta} \geq 0$$

$$X_B^{\alpha} \geq 0, \quad X_B^{\alpha} + (\theta - \alpha) P \geq 0$$

$$x_{B_i}^{\alpha} + (\theta-\alpha)\, p_i \geq 0, \quad i = 1(1)m.$$

At $\theta = \alpha$, $\quad x_{B_i}^{\alpha} \geq 0$.

$$d_j = x_{B_i}^{\alpha}, \quad \lambda = \theta - \alpha, \quad d_j' = p_i$$

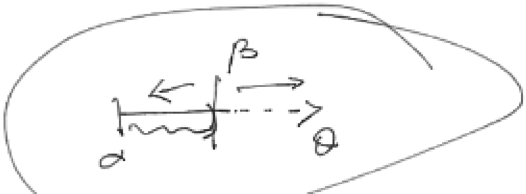
$$\max_{p_i > 0} \left[\frac{-x_{B_i}^{\alpha}}{p_i} \right] \leq (\theta - \alpha) \leq \min_{p_i < 0} \left[\frac{-x_{B_i}^{\alpha}}{p_i} \right]$$

Observation :—

i) $p_i > 0, \forall i$, x_B^{α} remains feasible for $\theta \geq \alpha$.

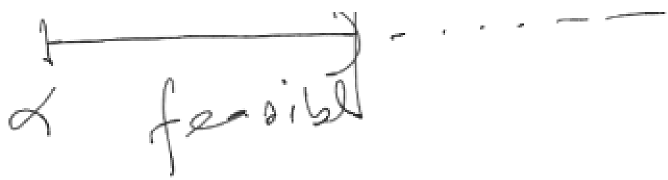
ii) If $p_i < 0$, for at least one i, there exist a critical value $\theta = \beta$

$$\beta - \alpha = \min_{p_i < 0} \left[\frac{-x_{B_i}^{\alpha}}{p_i} \right]$$



$$\beta = \alpha + \min_{p_i < 0} \left(\frac{-x_{B_i}^{\alpha}}{p_i} \right)$$

β Not feasible



	x_B
x_1	-20 → leaving vector
x_2	
x_4	-15