Problem:—

$3 \quad -6\theta$

$2 \quad -2\theta$

$(3,2,5,0,0,0) + \theta(-6,-2,5,0,0,0)$

$$\text{Max } z = (3-6\theta)x_1 + (2-2\theta)x_2 + (5+5\theta)x_3$$

s.t.,
$$x_1 + 2x_2 + x_3 \le 430$$
$$3x_1 + 2x_3 \le 460$$
$$x_1 + 4x_2 \le 420$$
$$x_1, x_2, x_3 \ge 0.$$

find the range of θ over which the solution remains B.F.S and optimal.

$\Rightarrow$ At $\theta = 0$

			3	2	5	0	0	0	
Bas. Var	C_B	x_B	x_1	x_2	x_3	x_4	x_5	x_6	
x_2	2	100	$-1/4$	1	0	$\boxed{1/2}$	$-1/4$	0	$\rightarrow$
x_3	5	230	$3/2$	0	1	0	$1/2$	0	
x_6	0	20	2	0	0	-2	1	1	
	$z = C_B x_B = 1350$		4	0	0	1 $\uparrow$	2	2	$\leftarrow \Delta_j^0$

$$c^\theta = c + \theta f = (3, 2, 5, 0, 0, 0) + \theta(-6, -2, 5, 0, 0, 0)$$

$$f = (-6, -2, 5, 0, 0, 0) = (f_1, f_2, f_3, f_4, f_5, f_6)$$

(x_2, x_3, x_6) basic var at $\theta = 0$.

$$c_B^\theta = c_B + \theta f_B = (2, 5, 0) + \theta(-2, 5, 0)$$

$$f_B = (-2, 5, 0)$$

$$\therefore \quad f_B x_1 - f_1 = (-2, 5, 0)\left(-\tfrac{1}{4}, \tfrac{3}{2}, 2\right) - (-6)$$
$$= 14 > 0$$

$$f_B x_2 - f_2 = (-2, 5, 0)(1, 0, 0) - (-2)$$

$$f_B x_2 - f_2 = (-2,5,0)(1,0,0) - (-2)$$
$$= 0$$
$$f_B x_3 - f_3 = 0$$
$$f_B x_4 - f_4 = (-2,5,0)(\tfrac{1}{2},0,-2) - 0$$
$$= -1$$
$$f_B x_5 - f_5 = 3$$
$$f_B x_6 - f_6 = 0.$$

Critical value θ' of θ after $\theta = 0$

$$\beta = \theta', \ \alpha = 0.$$

$$\theta' = 0 + \min_{\substack{j \\ f_B x_j - f_j < 0}} \left[\frac{-\Delta_j}{f_B x_j - f_j} \right]$$
$$= 0 + \frac{-\Delta_4}{f_B x_4 - f_4}$$
$$= \frac{-1}{-1} = 1$$

x_B^{θ} remains optimal for $0 \le \theta < \theta'$

i.e. $0 \le \theta < 1$

Determination of entering vector :—

$$z_j^{\theta'} - c_j^{\theta'} = (z_j^0 - c_j^0) + \theta'(f_B x_j - f_j)$$
$$\Delta_j^{\theta'} = \Delta_j^0 + \theta'(f_B x_j - f_j)$$

for non-basic variable,

$$\Delta_1^{\theta'} = \Delta_1^0 + \theta'(f_B x_1 - f_1)$$

$$= 4 + 1.(14) = 18$$

$$\Delta_4^{\theta'} = \Delta_4^{0} + \theta'(f_B x_4 - f_4)$$

$$= 1 + 1.(-1) = 0$$

$$\Delta_5^{\theta'} = \Delta_5^{0} + \theta'(f_B x_5 - f_5)$$

$$= 1 + 1.3 = 4.$$

$\Delta_4 = 0$ at $\theta' = 1$, we introduce x_4.

Leaving vector x_2.

Next optimal table for $\theta = 1$

		$c_j \rightarrow$	-3	0	10	0	0	0	
Bas var.	C_B	x_B	x_1	x_2	x_3	x_4	x_5	x_6	
x_4	0	200	$-1/2$	2	0	1	$-1/2$	0	
x_3	10	230	$3/2$	0	1	0	$1/2$	0	
x_6	0	420	1	4	0	0	0	1	
	$Z = C_B x_B = 2300$		18	0	0	0	5	0	$\leftarrow \Delta_j^{\theta'}$