# Parametric Linear Programming

$\underline{c}$, $\underline{b}$, $\underline{a}_{ij}$

$$c^{\theta} = c + \theta f = (c_1, c_2, \ldots, c_n) + \theta(f_1, f_2, \ldots, f_n)$$

**Type of linear variation :—**

$$\text{Max } z = cX, \quad \text{Sub to}, \quad AX = b, \; X \geq 0.$$

i) Linear variation in $\underline{c}$-vector.

ii) " " in b-vector.

iii) variations in the coeff. of the non-basic vector $a_j$.

iv) Simultaneous variation in c & b.

Let $\theta$ be the parameter of variation.
The linear functions of the above coeff may be defined as :—

$$\underline{c}^{\theta} = c + \theta f = (c_1, c_2, \ldots, c_n) + \theta(f_1, f_2, \ldots, f_n)$$

linear

and $$b^{\theta} = b + \theta g = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix} + \theta \begin{pmatrix} g_1 \\ g_2 \\ \vdots \\ g_m \end{pmatrix}$$

linear

$\theta$ is non neg.

Maximizing Problem.

**Linear variation in c-vector :—**

Let $\alpha \leq \theta \leq \beta$, $\alpha$ is arbitrary, algebraically small but finite no.

$\beta$ is arbitrary, algebraically large but finite no.

[illegible] no.

for each $\theta$ in this interval, find a sol$^n$ $(x_1, x_2, \ldots, x_n)$ to maximize

$$\left.\begin{aligned} z &= \sum_{j=1}^{n} (c_j + \theta f_j)\, x_j \\ \text{s.t.} \quad & \sum_{j=1}^{n} a_{ij} x_j = b_i, \quad i = 1(1)m \\ & x_j \geq 0, \quad j = 1(1)n. \end{aligned}\right\} \longrightarrow (1)$$

where, $c_j, f_j, a_{ij}, b_i$ are given constants.

## Solution Procedure :—

$\theta = 0$, solve the problem.

$\text{Max } z = cx, \quad Ax = b, \quad x \geq 0.$

(simplex $\rightarrow$ optimality, final table, $z$ optimal value)

$x_B = B^{-1} b$ unaltered

$$z_j - c_j \geq 0$$

for all non-basic vector $a_j$,

$$c_B x_j - c_j \geq 0.$$

Let $c = (c_1, c_2, \ldots, c_n)$

$$c^{\theta} = (c_1 + \theta f_1, c_2 + \theta f_2, \ldots, c_n + \theta f_n)$$

$$c^{\theta} = c + \theta f$$

$x_n^{\theta}$ the opt. sol$^n$ at $\theta = 0$.

Let $\theta = \alpha$ and $\theta = \beta$ be two consecutive critical values $(\alpha \geq \beta)$.

Opt. basic s.l^n at $\theta = \alpha$ is already determined is denoted by $x_B^{\alpha}$. Then we can determine the next critical value of $\theta$ :—

$$z_j^{\theta} - c_j^{\theta} \geq 0, \quad \forall j = 1(1)n \quad (\theta \geq \alpha)$$

$$c_j^{\theta} x_j - c_j^{\theta} \geq 0$$

$$\Rightarrow (c_B^{\theta} + \theta f_B) x_j - (c_j^{\theta} + \theta f_j) \geq 0$$

$$\Rightarrow \left[(c_B^{\theta} + \alpha f_B) + (\theta - \alpha) f_B\right] x_j - \left[(c_j^{\theta} + \alpha f_j) + (\theta - \alpha) f_j\right] \geq 0$$

$$\Rightarrow \left[c_B^{\alpha} + (\theta - \alpha) f_B\right] x_j - \left[c_j^{\alpha} + (\theta - \alpha) f_j\right] \geq 0$$

$$\left[\because \; c_B^{\theta} + \alpha f_B = c_B^{\alpha}, \quad c_j^{\theta} + \alpha f_j = c_j^{\alpha}\right]$$

$$\Rightarrow \left[c_B^{\alpha} x_j - c_j^{\alpha}\right] + (\theta - \alpha)\left[f_B x_j - f_j\right] \geq 0$$

$$\Rightarrow \left[ z_j^{\alpha} - c_j^{\alpha} \right] + (\theta - \alpha) \left[ f_B x_j - f_j \right] \geq 0$$

when $\theta = \alpha$, $z_j^{\alpha} - c_j^{\alpha} \geq 0$

Lemma 1,

$$\max_{\substack{j \\ p > 0}} \left[ \frac{-(z_j^{\alpha} - c_j^{\alpha})}{p} \right] \leq (\theta - \alpha)$$

$$\leq \min_{\substack{j \\ p < 0}} \left[ \frac{-(z_j^{\alpha} - c_j^{\alpha})}{p} \right] \qquad \text{(A)}$$

where $p = f_B x_j - f_j$

Next critical value $\theta = \beta$ is given by

$$\beta - \alpha = \min_{\substack{j \\ f_B x_j - f_j < 0}} \left[ \frac{-(z_j^{\alpha} - c_j^{\alpha})}{f_B x_j - f_j} \right]$$

$$\beta = \alpha + \min_{j} \left[ \quad " \quad \right]$$