Case 2 :— $a_{ij} \in B$

B change, $x_B = B^{-1} b$

z_j

$$\underset{\substack{k \neq p \\ p > 0}}{\text{Max}} \left[-\frac{x_{Bk}}{p} \right] \leq \Delta a_{ij} \leq \underset{\substack{k \neq p \\ p < 0}}{\text{Min}} \left[-\frac{x_{Bk}}{p} \right],$$

$$p = x_{Bk} \beta_{pi} - x_{Bp} \beta_{ki}$$

—— (1)

$$\underset{\substack{q \\ p > 0}}{\text{Max}} \left[\frac{-\Delta_q}{p} \right] \leq \Delta a_{ij} \leq \underset{\substack{q \\ p < 0}}{\text{Min}} \left[\frac{-\Delta_q}{p} \right],$$

$$\Delta_q = z_q - c_q$$

—— (2)

Ex :— The optimal solⁿ of the L.P.P.

$$\text{Max } Z = 10x_1 + 3x_2 + 6x_3 + 5x_4$$

S.t.

$$x_1 + 2x_2 + x_4 \leq 6$$

$$3x_1 + 2x_3 \leq 5$$

$$x_2 + 4x_3 + 5x_4 \leq 3$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

is given by the final table —

	$c_j \rightarrow$		10	3	6	5	0	0	0
Bas Var	C_B	x_B	x_1	x_2	x_3	x_4	S_1	S_2	S_3
x_2	3	$56/27$	0	1	$-22/27$	0	$5/9$	$-5/27$	$-1/9$
x_1	10	$5/3$	1	0	$2/3$	0	0	$1/3$	0
x_4	5	$5/27$	0	0	$26/27$	1	$-1/9$	$1/27$	$2/9$
	$Z = C_B x_B$		0	0	$\frac{82}{27}$	0	$\frac{10}{9}$	$\frac{80}{27}$	$7/9$

$= \frac{643}{27}$

Compute the limits for a_{11} so that the new sol^n remains optimal feasible.

$\Rightarrow$ Limit of Δa_{11}, $i=1$, $j=1$

$P = 2 \quad (a_1 = \beta_2)$

$x_{B1}\beta_{21} - x_{B2}\beta_{11} = \frac{56}{27} \times 0 - \frac{5}{3} \times \frac{5}{9}$

$= -\frac{25}{27} < 0$

$x_{B3}\beta_{21} - x_{B2}\beta_{31} = \frac{5}{27} \times 0 - \frac{5}{3} \times \left(-\frac{1}{9}\right)$

$= \frac{5}{27} > 0.$

From (1),

$-\frac{5}{27} \times \frac{27}{5} < \Delta a_{11} < -\frac{56}{27} \times \left(-\frac{27}{25}\right)$

$\Rightarrow -1 \leq \Delta a_{11} < \frac{56}{25}$ —— (A)

$c_B \beta_1 = \frac{10}{9}.$

$\beta_{21}\Delta_3 - x_{23} c_B \beta_1 = 0 - \frac{2}{3} \times \frac{10}{9} = -\frac{20}{27} < 0$

$\beta_{21}\Delta_5 - x_{25} c_B \beta_1 = 0 - 0 = 0$

$\beta_{21}\Delta_6 - x_{26} c_B \beta_1 = 0 - \frac{1}{3} \cdot \frac{10}{9} = -\frac{10}{27} < 0$

$\beta_{21}\Delta_7 - x_{27} c_B \beta_1 = 0 - 0 = 0.$

From (2),

$$-\infty \le \Delta a_{11} \le \min\left[-\frac{82}{27} \times -\frac{27}{20},\; -\frac{80}{27} \times -\frac{27}{10} \right]$$

$$-\infty \le \Delta a_{11} \le \frac{41}{10} \quad \text{———} \quad B$$

from A, B,

$$-1 \le \Delta a_{11} \le 56/25$$