Change in the component 'b_i' of vector b:-

$$b = (b_1, b_2, \ldots, b_l, \ldots, b_m)$$

Let b_l in changed.

$$\underline{X_B} = B^{-1} b \qquad \Delta_j = z_j - c_j \geq 0$$

Let b_l is changed to $b_l + \Delta b_l$

$$\hat{b} = (b_1, b_2, \ldots, b_l + \Delta b_l, \ldots, b_m)$$

To find the limits under which Δb_l can vary so that the feasibility of new solution $\hat{X}_B = B^{-1}\hat{b}$ is preserved and B remains the optimal basis.

$$\text{Let } B^{-1} = (\beta_1, \beta_2, \ldots, \beta_m)$$

$$= \begin{pmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1l} & \cdots & \beta_{1m} \\ \beta_{21} & \beta_{22} & \cdots & \beta_{2l} & \cdots & \beta_{2m} \\ \vdots & \vdots & & & & \vdots \\ \beta_{m1} & \beta_{m2} & \cdots & \beta_{ml} & \cdots & \beta_{mm} \end{pmatrix}$$

$$\hat{b} = (b_1, b_2, \ldots b_l, \ldots, b_m) + (0, 0, \ldots, \Delta b_l, \ldots, 0)$$

$$= b + (0, 0, \ldots, \Delta b_l, \ldots, 0)$$

$$\hat{X}_B = B^{-1}\hat{b} = \underline{B^{-1} b} + B^{-1}(0, 0, \ldots, \Delta b_l, \ldots 0)$$

$$\hat{X}_B = X_B + (\beta_1, \beta_2, \ldots, \beta_l, \ldots, \beta_m) \cdot (0, 0, \ldots \Delta b_l, \ldots 0)$$

$$= X_B + \beta_l \Delta b_l$$

$$\underline{\hat{X}_{Bi} = X_{Bi} + \beta_{il} \Delta b_l}$$

Since the feasibility of X_B is preserved, $(\hat{X}_B \geq 0)$

$$\left.\begin{array}{r} x_{Bi} + \beta_{il}\,\Delta b_l \geq 0 \\ x_{Bi} \geq 0 \end{array}\right\} \quad [x_B \geq 0]$$

$i = 1(1)m.$

$d_j = x_{Bi}$, $\lambda = \Delta b_l$, $d_j' = \beta_{il}$ in lemma,

$$\underset{\substack{i \\ \beta_{il} > 0}}{\text{Max}} \left[-\frac{x_{\beta i}}{\beta_{il}}\right] \leq \Delta b_l \leq \underset{\substack{i \\ \beta_{il} < 0}}{\text{Min}} \left[-\frac{x_{\beta i}}{\beta_{il}}\right]$$

Problem:—

$$\text{Max } Z = 3x_1 + 5x_2 + 4x_3$$

s.t.

$$2x_1 + 3x_2 \leq 8$$

$$2x_2 + 5x_3 \leq 10$$

$$3x_1 + 2x_2 + 4x_3 \leq 15$$

$$x_1, x_2, x_3 \geq 0.$$

Find the range of b_2 can be changed maintaining the feasibility of the solution.

$\Rightarrow$

Optimal table

	c_j		3	5	4	0	0	0
Basic var.	C_B	X_B	x_1	x_2	x_3	x_4	x_5	x_6
x_2	5	50/41	0	1	0	15/41	8/41	-10/41
x_3	4	62/41	0	0	1	-6/41	5/41	4/41
[illegible]	3	89/41	1	0	0	-2/41	-12/41	15/41

$z = \frac{765}{41}$	0	0	0	$45/41$	$24/41$	$11/41$	Δ_j

$$B = (x_2, x_3, x_1), \quad x_B = \left(\frac{50}{41}, \frac{62}{41}, \frac{89}{41}\right)$$

$$b = (8, 10, 15)$$

$$B^{-1} = \begin{pmatrix} \frac{15}{41} & \frac{8}{41}\ \beta_{12} & \frac{10}{41} \\ -\frac{6}{41} & \frac{5}{41}\ \beta_{22} & \frac{5}{41} \\ -\frac{2}{41} & -\frac{12}{41}\ \beta_{32} & \frac{15}{41} \end{pmatrix}$$

$$\Delta b_l = \Delta b_2 .$$

$$\underset{\beta_{i2} > 0}{\underset{i}{\text{Max}}} \left[\frac{-x_{B1}}{\beta_{12}}, \frac{-x_{B2}}{\beta_{22}} \right] \leq \Delta b_2 \leq \underset{\beta_{i2} < 0}{\underset{i}{\text{Min}}} \left[\frac{-x_{B3}}{\beta_{32}} \right]$$

$$\Rightarrow \underset{\beta_{i2} > 0}{\underset{i}{\text{Max}}} \left[\frac{-\frac{50}{41}}{\frac{8}{41}}, \frac{-\frac{62}{41}}{\frac{5}{41}} \right] \leq \Delta b_2 \leq \underset{\beta_{i2} < 0}{\underset{i}{\text{Min}}} \left[\frac{-\frac{89}{41}}{-\frac{12}{41}} \right]$$

$$\Rightarrow \text{Max}\left[-\frac{50}{8}, -\frac{62}{5} \right] \leq \Delta b_2 \leq \frac{89}{12}$$

$$\Rightarrow -\frac{25}{4} \leq \Delta b_2 \leq \frac{89}{12}$$