$$z = cx$$
$$Ax = b$$

Change in the component 'a_{ij}' of A :—

Possibilities :—

i) a_{ij} is changed to $a_{ij} + \Delta a_{ij}$ which does not belong to the basis matrix B.

ii) a_{ij} is changed to $a_{ij} + \Delta a_{ij}$ belongs to B.

Case I :—

$$a_{ij} \notin B.$$

a_{ij} does not affect the optimal solⁿ,

$$X_B = B^{-1} b$$

a_{ij} can effect on $\Delta_j = Z_j - c_j \ (\geq 0)$

Let $B^{-1} = [\beta_1, \beta_2, \ldots, \beta_m]$, $a_j = [a_{1j}, a_{2j}, \ldots \ldots a_{ij}, \ldots, a_{mj}]$

a_{ij} is changed to $a_{ij} + \Delta a_{ij}$.

$$\hat{a}_j = [a_{1j}, a_{2j}, \ldots, a_{ij} + \Delta a_{ij}, \ldots, a_{mj}]$$
$$= a_j + [0, 0, \ldots, \Delta a_{ij}, \ldots, 0]$$
$$\hat{Z}_j = c_B B^{-1} \hat{a}_j$$
$$= c_B B^{-1} [a_j + (0, 0 \ldots \Delta a_{ij}, \ldots 0)]$$
$$= c_B B^{-1} a_j + c_B B^{-1} (0, 0, \ldots, \Delta a_{ij}, \ldots 0)$$
$$= Z_j + c_B (\beta_1, \beta_2, \ldots, \beta_m)(0, 0, \ldots \Delta a_{ij}, \ldots$$

$$= z_j + c_B \beta_i \Delta a_{ij}$$

The optimality condition will remain unaltered if

$$z_j - c_j \geq 0, \quad (z_j + c_B \beta_i \Delta a_{ij}) - c_j \geq 0$$

$$z_j - c_j \geq 0, \quad (z_j - c_j) + (c_B \beta_i) \Delta a_{ij} \geq 0$$

By lemma 1,

$$\max_{\substack{i \\ c_B \beta_i > 0}} \left[\frac{-\Delta_j}{c_B \beta_i} \right] \leq \Delta a_{ij} \leq \min_{\substack{i \\ c_B \beta_i < 0}} \left[\frac{-\Delta_j}{c_B \beta_i} \right] \longrightarrow (A)$$

$$[\Delta_j = z_j - c_j]$$

Δa_{ij} is unrestricted if $c_B \beta_i = 0$.

Problem :-

Example - 1

$a_4 \notin B$, $B = (x_2, x_3, x_1)$.

$$B^{-1} = (\beta_1, \beta_2, \beta_3) = \begin{pmatrix} 15/41 & 8/41 & -10/41 \\ -6/41 & 5/41 & 4/41 \\ -2/41 & -12/41 & 15/41 \end{pmatrix}$$

$$C_B = (5 \quad 4 \quad 3)$$

$$c_B \beta_1 = (5 \quad 4 \quad 3) \left(15/41 \right.$$

$$\begin{pmatrix} -6/41 \\ -2/41 \end{pmatrix}$$

$$= \frac{45}{41} \quad \underline{(z_4 - c_4)}$$

$$c_B \beta_2 = (5 \;\; 4 \;\; 3) \begin{pmatrix} 8/41 \\ \frac{5}{41} \\ -12/41 \end{pmatrix}$$

$$= 24/41 \quad (z_5 - c_5)$$

$$c_B \beta_3 = 11/41 \quad (z_6 - c_6)$$

From A,

$$\frac{-\Delta_4}{c_B \beta_1} \le \Delta a_{14} < \infty \quad (\because c_B \beta_i > 0)$$

$$\Rightarrow -\frac{45/41}{45/41} \le \Delta a_{14} < \infty$$

$$\Rightarrow -1 \le \Delta a_{14} < \infty$$

Similarly, $-1 \le \Delta_{24} < \infty$

$$-1 \le \Delta_{34} < \infty$$