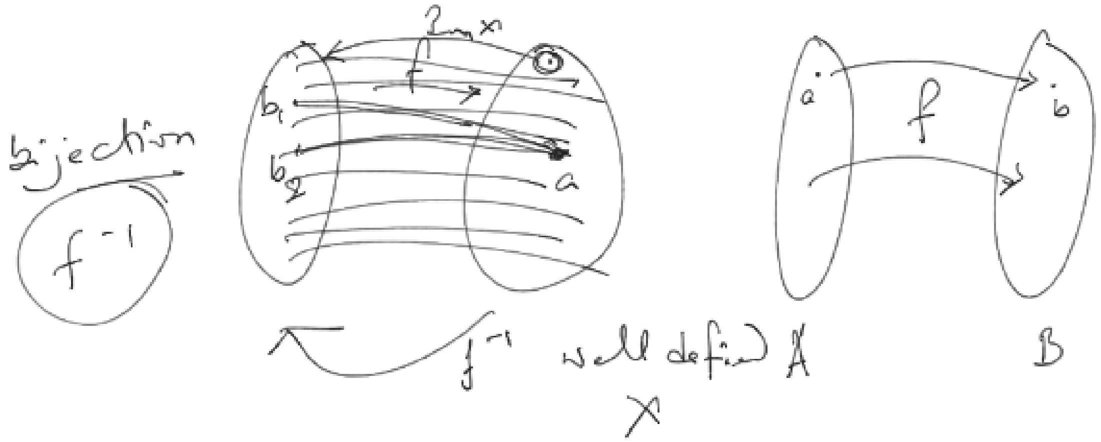
# Binary operation

$$S \times S \longrightarrow S$$
$$(a, b) \longrightarrow a \circ b \in S.$$




**Def$^n$** :— Let $S$ be any nonempty set. A binary operation on $S$ is a function that assigns each ordered pair of elements of $S$ to an elements of $S$.

'$*$' on a set $S$ will be a binary operator if

i) Exactly one element is assigned to each possible ordered pair of elements of $S$.

$a * b = c$ $\forall a, b$

ii) for each ordered pair of elements of $S$, the element assigned to it belongs to $S$.

\# **Def** :— A binary operation of a set $S$ is a function $S \times S$ to $S$. For each $(a, b) \in S \times S$, we have $a \circ b$ in $S$. [ '$\circ$' is the binary operator ]

/ a) $S$ is closed under '$\circ$'.

ii) $(S, \circ)$ is closed.

Ex:— i) $(\mathbb{R}, +)$ is closed.

'$+$' is a binary operation on $\mathbb{R}$.

ii) $(\mathbb{R}^*, +)$ $\mathbb{R}^* \equiv \mathbb{R} - \{0\}$.

$-n + n = 0 \notin \mathbb{R}^*$.

'$+$' is not binary on $\mathbb{R}^*$.

iii) $M(\mathbb{R})$, the set of all real matrices.

Usual matrix addition is <u>not</u> binary as $A + B$ is not defined when $A$ and $B$ has different orders.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix}$$

<u>Induced operation</u> :—

$(G, *)$, $H \subseteq G$.

Let $*$ be a binary operation on $G$ $[* : G \times G \to G]$ and $H$ be a subset of $G$.

$H$ is closed under $*$ if $a * b \in H$, $\forall\, a, b \in H$.

In this case, the binary operation on $H$ (by restricting $*$ on $H$) is the induced operation on $H$.

Ex:— 1) $(\mathbb{R}, +)$, $\mathbb{Z} \subset \mathbb{R}$.

$(\mathbb{Z}, +)$ is closed. '$+$' is induced on $\mathbb{Z}$.

2) $(\mathbb{R}, +)$ $\mathbb{R}^* \subset \mathbb{R}$

2) $(\mathbb{R}^*, +)$, $\mathbb{R}$

$(\mathbb{R}^*, +)$ is not closed. '+' is not induced on $\mathbb{R}^*$.

3) $H \subseteq \mathbb{Z}^+$, $H = \{n^2 : n \in \mathbb{Z}^+\}$

$(H, +)$ is not closed.

$(H, \cdot)$ is closed.

$a, b \in S$, $a_1, b_1 \in S$

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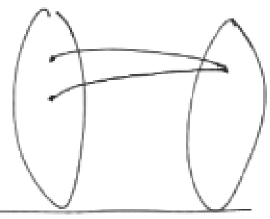
$(a, b) \to a \circ b$ (unique)

$(a_1, b_1) \to a_1 \circ b_1$ (unique)



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$A$ = Math 2nd year = $\{a_1, a_2, \ldots, a_{50}\}$

$(A, *)$ age of students are diff.

$a_1 * a_2 = c$, $c$ is the youngest but older than both $a_1$ and $a_2$.

$*$ is Not binary.

Take $a_i$ and $a_j$, the oldest and second oldest.

$a_i * a_j$ is not defined.

Ex:—

1) $\mathbb{Z}^+$. Define a binary operator $*$ on $\mathbb{Z}^+$ by

$a * b$ = smaller of $a$ & $b$ or the common value if $a = b$.

$7 * 23 = 7$, $11 * 11 = 11$.

2) $(Z^+, *')$ by $a *' b = a$

$7 *' 23 = 7$, $23 *' 7 = 23$.

3) $(Z^+, *'')$, by $a *'' b = (a * b) + 2$

[ $*$ is defined in ex 1]

$7 *'' 23 = (7 * 23) + 2 = 7 + 2 = 9$.

Commutative :—

Ex 1, 3 are commutative.

Ex 2 is not ".

Associative :—

Ex 2 $\longrightarrow$ $a *' (b *' c) = a *' b$
$= a$

$(a *' b) *' c = a *' c = a$.

Ex 1 associative  Associative

Ex 3 :— $(2 *'' 5) *'' 9 = 4 *'' 9 = 6$

$2 *'' (5 *'' 9) = 2 *'' 7 = 4$

Not Associative.

Def$^n$ :— 1) A binary operation $*$ on $S$ is commutative iff $a * b = b * a$, $\forall a, b \in S$.

2) A binary operation $*$ on $S$ is associative iff $a * (b * c) = (a * b) * c$. $\forall a, b, c \in S$.