	R_0	R_{90}	R_{180}	R_{270}	H	V	D	D'
R_0	R_0	R_{90}	R_{180}	R_{270}	H	V	D	D'
R_{90}	R_{90}	R_{180}	R_{270}	R_0	D'	D	H	V
R_{180}	R_{180}	R_{270}	R_0	R_{90}	V	H	D'	D
R_{270}	R_{270}	R_0	R_{90}	R_{180}	D	D'	V	H
H	H	D	V	D'	R_0	R_{180}	R_{90}	R_{270}
V	V	D'	H	D	R_{180}	R_0	R_{270}	R_{90}
D	D	V	D'	H	R_{270}	R_{90}	R_0	R_{180}
D'	D'	H	D	V	R_{90}	R_{270}	R_{180}	R_0

Cayley table

$X = \{R_0, R_{90}, R_{180}, R_{270}, H, V, D, D'\}$

1) Let $A, B \in X \Rightarrow A \cdot B \in X$, $\forall A, B \in X$.

(closure prop)

X is closed.

[$Y = \{1, 2, 3, 4\} \subseteq \mathbb{N}$.

$(Y, +) \Rightarrow 2 + 3 = 5 \notin Y$ (Not closed)]

$+$, $\cdot$ operator (binary)

2) Let $A, B, C \in X$

$A \cdot (B \cdot C) = (A \cdot B) \cdot C$ [Associative prop]

$f \circ (g \circ h) = (f \circ g) \circ h$

[Functional composition is associative]

8^3 (Heredity prop)

3) For any $A \in X$,

$$A \cdot R_0 = A = R_0 \cdot A$$ (Identity prop)

$$H \cdot R_0 = H = R_0 \cdot H$$

R_0 is the identity element.

4) $$R_0 \cdot R_0 = R_0$$

$$R_{90} \cdot R_{270} = R_0 = R_{270} \cdot R_{90}$$

$$R_{180} \cdot R_{180} = R_0 \Rightarrow R_{180} = R_{180}^{-1}$$

$$H \cdot H = R_0 = V \cdot V = D \cdot D = D' \cdot D'$$

$$R_{90} = R_{270}^{-1}, \quad R_{270} = R_{90}^{-1}$$

Rest elements are self-inverse.

$$(\mathbb{R}^*, \cdot) \qquad A \cdot B = I_n = B \cdot A$$

$$B = A^{-1}$$

$$a \cdot b = 1$$

$$\Rightarrow b = \frac{1}{a} = a^{-1}$$ (Inverse prop)

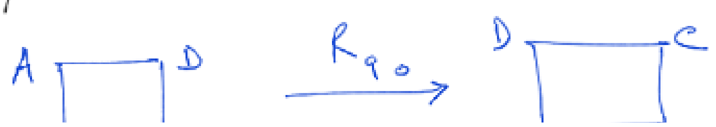
5) $$A \cdot B = B \cdot A, \quad \forall A, B$$ (Commutative prop)

$$V \cdot R_{90} = D'$$

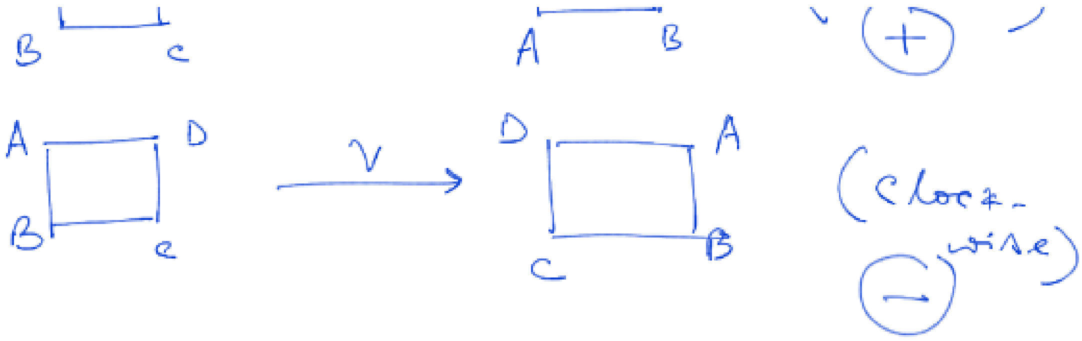
$$R_{90} \cdot V = D \qquad V \cdot R_{90} \neq R_{90} \cdot V$$

$\therefore$ $\times$ is not commutative.

	Rot	Ref
Rot	Rot	Ref
Ref	Ref	Rot

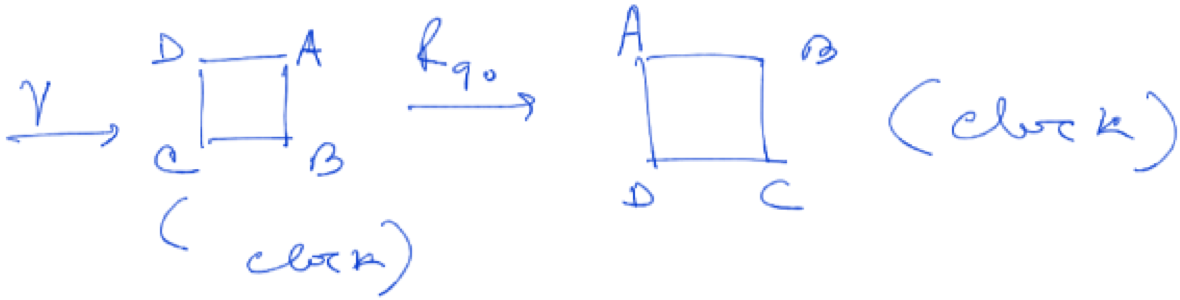


(anti-clock wise)



$V R_{90} \rightarrow$ clockwise (Ref)

$R_{90} V \rightarrow$ clockwise (Ref)



$V D' = R_{90}$, $D' V = R_{270}$