Beta Radioactivity

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References

- Nuclear Physics by I. Kaplan
- Concepts of Modern Physics by A. Beiser
- Nuclear Physics by S. N. Ghoshal

Introduction

- Experimentally it was detected that beta particles have the same mass of electron i.e. 0.511 MeV. It can be of $\pm e$ charge; the former is β^+ and the later one is β^-.

- Unlike alpha particles, beta disintegration can be observed for lighter nuclei also.

- Detail understanding of beta disintegration leads to manifold aspects of nuclear and particle physics
  - Idea of neutrino and associated physics of neutrino.
  - Details of nuclear energy spectra.
  - Theory of beta decay and idea of weak interaction.

  and so on.....

Beta emission

- β^- decay:

$$^A_ZX \rightarrow \ ^{\ \ A}_{Z+1}Y + \beta^-$$

- β^+ decay:

$$^A_ZX \rightarrow \ ^{\ \ A}_{Z-1}Y + \beta^+$$

- There exist another type of beta activity, called electron capture process in which neither β^- nor β^+ are emitted. But, atomic electron (mostly K-shell) electron is captured by nucleus and the process is detected via X-ray emission.

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- **Note:**
The above mentioned two processes of $\beta^\pm$ decay is not complete. Later we will see that these process violate some conservation principle.

5.1. Exercise

Suppose electron can exist inside the nuclei and which are emitted as β^- particles.

(a) Estimate the nuclear diameter from $R = r_0 A^{1/3}$ with $r_0 \approx 1.2\ fm$ and take $A \sim 100$.

(b) Use uncertainity relation to get Δp of electron inside nucleus and then calculate its KE assuming $p \sim \Delta p$ (do relativistic calculation).

(c) Compare the calculated KE with experimentally observed β^- KE (which is $\sim$ few MeV at most.) Hence, draw some conclusions about existence of electrons inside the nucleus.

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- **Example:**

$$^{135}_{52}\mathrm{Te} \xrightarrow{\beta^-} {}^{135}_{53}\mathrm{I} \xrightarrow{\beta^-} {}^{135}_{54}\mathrm{Xe} \xrightarrow{\beta^-} {}^{135}_{55}\mathrm{Cs} \xrightarrow{\beta^-} {}^{\mathbf{135}}_{\mathbf{56}}\mathbf{Ba} \xleftarrow{\mathrm{EC}} {}^{135}_{57}\mathrm{La} \xleftarrow{\beta^+} {}^{135}_{58}\mathrm{Ce}$$

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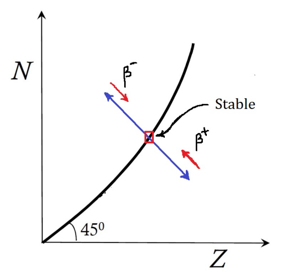

- All of these nuclei lie on a single isobaric line ($N + Z = const.$). Higher Z elements approaches to stable pt. from right via β^+ decay (or EC) and lower Z elements approaches to stable pt. from left via β^+ decay

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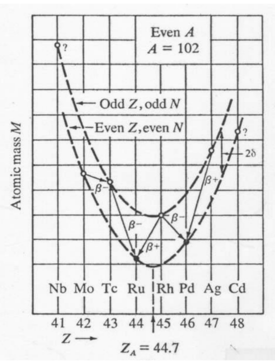

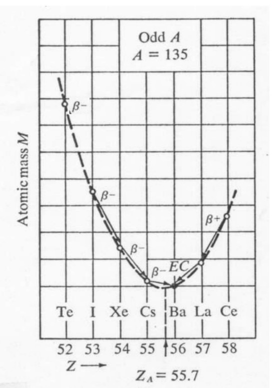

- **8.1. Exercise :**
  From liquid drop model explain the mass parabola and estimate the stable nuclei in terms of liquid drop model parameters. Obtain Z value of the stable nucleus for the isobars of $A = 135$ and compare it with the experimental findings (prev. example of mass parabola.)
- **8.2. Exercise :**
  In the slide-7, there are two parabolas for even A nuclei and one parabola for odd A nuclei. Explain this feature.

▶ 9.1. Exercise :

Atomic masses ($M(A,Z)$ in 'u' unit) of a set of **isobars** are given below.

A	Z	Element	M(A,Z) in u
135	52	TI	134.916448592
135	53	I	134.910048121
135	54	Xe	134.907227495
135	55	Cs	134.905977008
135	56	Ba	134.905688591
135	57	La	134.906976844
135	58	Ce	134.909151396
135	59	Pr	134.913111745
135	60	Nd	134.91818116

(**a**) Identify the most stable element (**b**) Plot $M(A,Z)$ as a function of Z and interpret the curve in context of $\beta^{\pm}$ decay and electron capture process. (**c**) Which element of this list will emit β^{-} particle of highest kinetic energy? (**d**) Represent these isobars in $N-Z$ stability curve. (**e**) Calculate nuclear binding energy per nucleon for each of the element (ignore binding energies of atomic electrons) and draw the binding energy per nucleon as a function of Z.
(Given : $m_e = 0.511$ MeV, $m_p = 938.280$ MeV, $m_n = 939.573$ MeV, $1\ u = 931.50$ MeV).

Q-value

- Beta activity is exothermic process. Energy released due to beta decay can be given by mass defect.
- For $\beta^{\pm}$ decay : ${}^{A}_{Z}\mathrm{X} \rightarrow {}_{Z\mp 1}^{\ \ A}\mathrm{Y} + \beta^{\pm}$
- $Q_{\beta^-} = [M_n(A, Z) - M_n(A, Z+1) - m_e]c^2$
  and
  $Q_{\beta^+} = [M_n(A, Z) - M_n(A, Z-1) - m_e]c^2$

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- It is more convenient to use atomic mass (M_{at}) instead of nuclear mass (M_n) to obtain the $Q-$values.
- If $B_i(X)$ be the binding energy of i-th atomic electron to nucleus of ${}^{A}_{Z}\text{X}$, then the obvious relation is,

$$M_{at}(A,Z) = M_n(A,Z) + Zm_e - \frac{1}{c^2}\sum_{i=1}^{Z} B_i(X)$$

- **11.1. Exercise :**
  Explain the relation of atomic mass and nuclear mass as written in the previous slide.

- **11.2. Exercise :**
  Show that,

$$Q_{\beta^-} = [M_{at}(A,Z) - M_{at}(A,Z+1)]c^2 + \left(\sum_{i=1}^{Z} B_i(X) - \sum_{i=1}^{Z+1} B_i(Y) \right)$$

  Hence justify that one can ignore safely the contributing term from binding energies of atomic electrons.

- **11.3. Exercise :**
  In this similar way obtain an similar expression of Q_{β^+} and show that β^+ disintegration is possible only when the atomic mass of mother nucleus is greater than daughter nucleus by at least of two electronic mass.

- Electron capture (EC) process is already mentioned; in this process one K-shell electron is absorbed to nucleus and as a consequence a proton is converted into neutron.

  The EC process : ${}^{A}_{Z}\mathrm{X} + e^- \rightarrow {}_{Z-1}^{\ \ A}\mathrm{Y}$

- During the EC process;

  Initial state $\equiv$ Bound state of ${}^{A}_{Z}\mathrm{X}$ nucleus and one K-shell electron.

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  Therefore,
  Initial mass of the system $= M_n(A, Z) + m_e - B_K(X)/c^2$
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  Therefore,
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- $Q_{EC} = [M_n(A, Z) + m_e - M_n(A, Z-1)]c^2 - B_K(X)$

- **13.1. Exercise :**
  For electron capture process, show that atomic mass of the mother nucleus must be greater than that of daughter nucleus by at least of K-shell electron binding energy.

- **13.2. Exercise :**
  ${}^{80}_{35}Br$ transforms to ${}^{80}_{34}Se$ by radioactive process. Br atom is heavier than Se atom by 2.66 MeV. What types of radioactivity is expected to be shown by Br. Explain your answer.

- **13.3. Exercise :**
  ${}^{7}_{4}Be$ atom is heavier than ${}^{7}_{3}Li$ atom by 0.864 MeV. What type of radioactivity is expected from Be to convert into Li ? Explain your answer.

- **14.1. Exercise**

The atomic masses of some pair of nuclei are given below (within first bracket) in atomic mass unit (amu)

(A) ${}^{7}_{3}\mathrm{Li}$ (7.0182) and ${}^{7}_{4}\mathrm{Be}$ (7.0192)
(B) ${}^{13}_{6}\mathrm{C}$ (13.0076) and ${}^{13}_{7}\mathrm{N}$ (13.0100)
(C) ${}^{19}_{9}\mathrm{F}$ (19.0045) and ${}^{19}_{10}\mathrm{Ne}$ (19.0080)
(D) ${}^{34}_{15}\mathrm{P}$ (33.9983) and ${}^{34}_{16}\mathrm{S}$ (33.9978)
(E) ${}^{35}_{16}\mathrm{S}$ (34.9791) and ${}^{35}_{17}\mathrm{Cl}$ (34.9789)

Identify and explain which kind of beta radioactivity (β^- or β^+ or electron capture) can be observed for each pair of nuclei (Given : m_e =0.00055 amu).

- **14.2. Exercise**

${}^{34}\mathrm{Cl}$ positron-decays to ${}^{34}\mathrm{S}$. Atomic mass difference of neutral ${}^{34}\mathrm{Cl}$ and ${}^{34}\mathrm{S}$ is 5.52 MeV/c^2, what is the maximum positron energy? (Ignore the energy of recoiled daughter nucleus, which is a fair approximation.)

Kinetic energy and Q-value

- Energy generated due to beta decay supplies kinetic energies to beta particle and recoiled daughter nucleus.

$$^{A}_{Z}X \rightarrow \, _{Z\mp1}^{\;\;A}Y + \beta^{\pm}$$

- Let simplify the notations: Q_β denotes Q-value for $\beta^{\pm}$ M_X, M_Y represent nuclear masses respectively.

$$Q_\beta = (M_X - M_Y - m_e)c^2 = T_Y + T_\beta$$

- From linear momentum conservation;

$$p_Y = p_\beta$$

- From this above information one can find out KE of recoiled daughter nucleus in terms of beta particle KE and it would be observed that recoiled daughter nucleus posses negligible amount of KE compared to beta particle KE.

▶ 16.1. Exercise

Consider the beta decay process: $X \rightarrow Y + \beta$

(a) Use relativistic KE of beta particle : $T_\beta = \sqrt{p_\beta^2 c^2 + m_e^2 c^4} - m_e c^2$ to obtain $p_\beta^2 c^2 = T_\beta (T_\beta + 1)$ in MeV unit (consider electronic rest mass = 0.5 MeV).

(b) Use linear momentum conservation $p_\beta = p_Y$ to obtain;

$$T_Y = \frac{T_\beta (T_\beta + 1)}{2 M_Y c^2} \quad ; \quad \text{in MeV unit}$$

(c) Estimate KE of of recoiled daughter nucleus of a typical beta decay process of $T_\beta \sim 1\ MeV$ and atomic number of mother nucleus ~ 10. Hence, conclude that $Q_\beta = T_\beta + T_Y \approx T_\beta$

Energy spectrum

- We can safely ignore the KE of recoiled daughter nucleus (see the previous exercise). So, $T_{\beta^\pm} \approx Q_{\beta^\pm} = [M_n(A,Z) - M_n(A, Z \mp 1) - m_e]c^2$.

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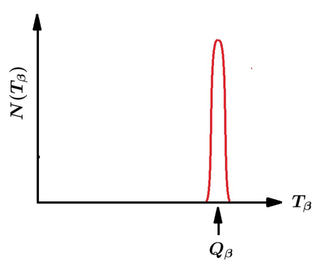
- $N(T_\beta)$ is the nos. of beta particles having KE within dT_β at T_β

Expected spectra



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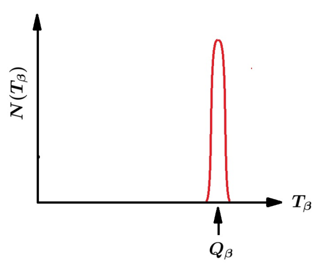
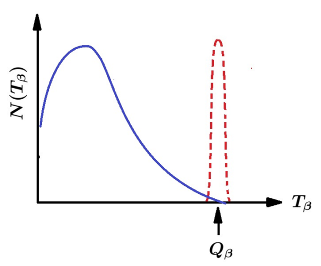


Observed spectra



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- When trajectory of beta particle and recoiled daughter nucleus is observed, they are not exactly opposite to each other. One can consider apparent violation of linear momentum conservation.

Neutrino hypothesis

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- Beta decay is always associated with neutrino emission. Neutrino shares energy with beta particle. $T_\nu + T_\beta = Q_\beta$.
- Considering neutrino and beta decay as simultaneous event, all the apparent violation of conservation principles can be explained.

- β^- decay : ${}^{A}_{Z}\mathrm{X} \rightarrow {}_{Z+1}^{\;\;A}\mathrm{Y} + e^- + \bar{\nu}_e$ or $n \rightarrow p + e^- + \bar{\nu}_e$
- β^+ decay : ${}^{A}_{Z}\mathrm{X} \rightarrow {}_{Z-1}^{\;\;A}\mathrm{Y} + e^+ + \nu_e$ or $p \rightarrow n + e^+ + \nu_e$
- electron capture : ${}^{A}_{Z}\mathrm{X} + e^- \rightarrow {}_{Z-1}^{\;\;A}\mathrm{Y} + \nu_e$ or $p + e^- \rightarrow n + \nu_e$

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- electron capture : ${}^A_Z\text{X} + e^- \rightarrow {}_{Z-1}^{\ \ A}\text{Y} + \nu_e$ or $p + e^- \rightarrow n + \nu_e$
- **20.1. Exercise**
  Given that $m_e = 0.511$ MeV, $m_p = 938.280$ MeV and $m_n = 939.573$ MeV. Hence show that free neutron can exhibit β^- decay but free proton can not exhibit β^+ decay.
- **20.2. Exercise**
  Write down the equations for inverse beta decays and explain why these events have extremely low probabilities.