Dynamical system, MSc, Semester III

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Highlights

1. Examples

2. Definition

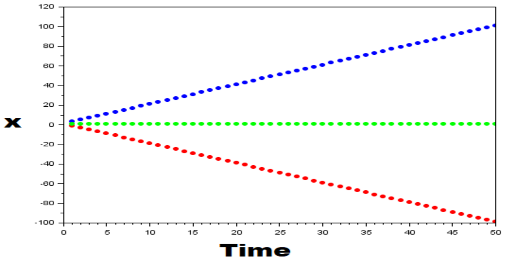
Example 1

Consider the simple initial value *ODE*

$$\frac{dx}{dt} = a, \quad x = x_0, \text{ when } t = 0.$$

Its solution is given by

$$x = at + x_0$$

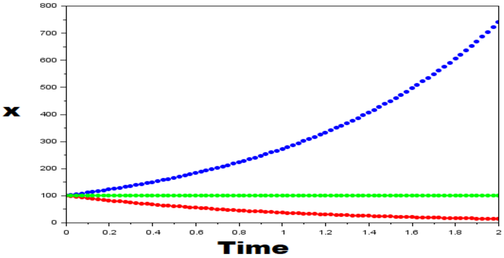


If the simple initial value *ODE* have the following form

$$\frac{dx}{dt} = ax, \quad x = x_0, \text{ when } t = 0.$$

Then its solution is given by

$$x = x_0 e^{at}$$



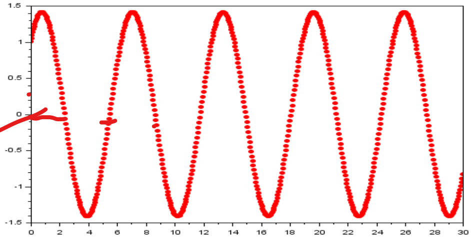
Example 2

Consider the simple initial value *ODE* (SHM)

$$\frac{d^2x}{dt^2} = -ax, \quad x(0) = x_0, \text{ and } \dot{x}(0) = x_1.$$

Its solution is given by

$$x = c_1 \cos \omega t + c_2 \sin \omega t$$



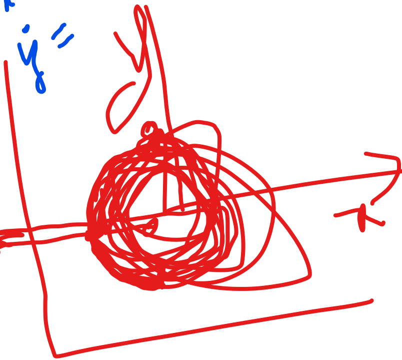
Let $\dot{x} = y$, then SHM problem can be written as

$$\dot{x} = y$$
$$\dot{y} = -ax$$

$\dot{x} =$

$\dot{y} =$

Can you find the solution??



Let $\underset{\sim}{x} = \underset{\sim}{x}(t) \in \mathbb{R}^n$, $t \in I \subseteq \mathbb{R}$ be the vector representing the dynamics of a continuous system (continuous-time system). The mathematical representation of the system may be written as

$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$\dot{x}_1 = f_1(x_1, x_2)$

$$\frac{\mathrm{d}\underset{\sim}{x}}{\mathrm{d}t} = \dot{\underset{\sim}{x}} = \underset{\sim}{f}(\underset{\sim}{x}, t)$$

where $f(\underset{\sim}{x}, t)$ is a sufficiently smooth function defined on some subset $U \subset \mathbb{R}^n \times \mathbb{R}$. Schematically, this can be shown as

$$\underset{(\text{state space})}{\mathbb{R}^n} \times \underset{(\text{time})}{\mathbb{R}} = \underset{(\text{space of motions})}{\mathbb{R}^{n+1}}$$

The variable t is usually interpreted as time and the function $f(\underset{\sim}{x}, t)$ is generally nonlinear. The time interval may be finite, semi-finite or infinite.

Continuous dynamical systems
Discrete dynamical systems

$x_{n+1} = f(x_n)$

Auto

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